Test 2: Algorithms

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1) (15 Points) Find the LCS For abaababaab and bababababab, give the table.

∅	a	b	a	a	b	a	b	a	a	b
b	0	↖1	←1	←1	↖1	←1	↖1	←1	←1	↖1
a	↖1	↑1	↖2	↖2	←2	↖2	←2	↖2	↖2	←2
b	↑1	↖2	↑2	↑2	↖3	←3	↖3	←3	←3	↖3
a	↖1	↑2	↖3	↖3	↑3	↖4	←4	↖4	↖4	←4
b	↑1	↖2	↑3	↑3	↖4	↑4	↖5	←5	←5	↖5
a	↖1	↑2	↖3	↖4	↑4	↖5	↑5	↖6	↖6	←6
b	↑1	↖2	↑3	↑4	↖5	↑5	↖6	↑6	↑6	↖7
a	↖1	↑2	↖3	↖4	↑5	↖6	↑6	↖7	↑7	↑7
b	↑1	↖2	↑3	↑4	↖5	↑6	↖7	↑7	↑7	↖8

ababa

2) (15Points) Use KMP to find if abaaba is a substring of abababbbaaababbaa. 18 characters
   a. Give π
   b. Show each step of the KMP algorithm.

a. P(1)=0 P(2)=0 P(3)=1 P(4)=2 P(5)=3 P(6)=4 P(7)=5 P(8)=0 P(9)=1 P(10)=1 P(11)=1 P(12)=2 P(13)=3 P(14)=4 P(15)=0 P(16)=1 P(17)=1

b.

```
* a b a b a b a b b a a a b a b b a a
  a b a a b a
    a b a a b a
      a b a a b a
        a b a a b a
          a b a a b a
            a b a a b a
              a b a a b a
                a b a a b a
                  a b a a b a
                    a b a a b a ← past length of string
```

substring does not exist

3) (20 Points) Fast Fourier Transforms

$A(x) = 2x - 1$

$B(x) = 3x+2$

$C(x)=A(x)*B(x) = 6x^2 + x - 2$

Calculate the coefficients of C(x), i.e. 6, 1, -2 by doing the following

a) (Evaluation) Change A(x) and B(x) to point-value representation using x=1, x=2, x=3
b) (Pointwise Multiplication) List the three points that must be found on C(x)
c) Let $C(x)= c_2x^2+c_1x+c_0$ and list three equations using b)
d) (Interpolation) Solve the three equations (Gaussian elimination), to get the coefficients.

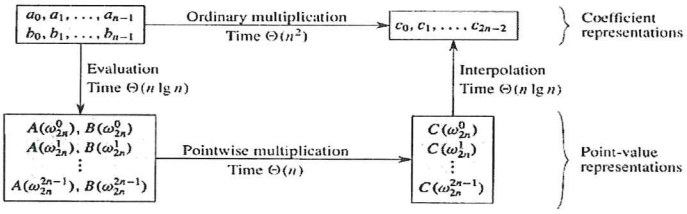


Figure 30.1 A graphical outline of an efficient polynomial-multiplication process. Representations on the top are in coefficient form, while those on the bottom are in point-value form. The arrows from left to right correspond to the multiplication operation. The ω_{2n} terms are complex $(2n)$th roots of unity.

- What is the running time for the most efficient version of FFT?

  $O(n \log n)$

- Explain the history behind FFT? Who was the first person, second person and third person to discover it? At what point did it become well-known?

  Gauss first used it to project the trajectories of 2 asteroids, then Tukey postulated detecting nuclear weapons test in the USSR. Tukey was put in touch with Cooley, who used it for analyzing crystallographic data, and FFTs became widely adopted after Tukey & Cooley published their findings. (Wikipedia)

4) (10 Points) Explain the advantages of Dynamic Programming over Divide and Conquer.

Dynamic programming can be used where divide and conquer cannot be implemented. For example, given a graph, it is not feasibly possible to implement a divide and conquer algorithm to find the shortest path, but dynamic programming can be used.

3. a + b

x	$2x-1$	$3x+2$	
0	-1	2	-2
1	1	5	5
-1	-3	-1	3

c. $(0,-2)$ $\quad -2 = c_2 0^2 + c_1 0 + c_0 \rightarrow -2 = c_0$

$(1,5)$ $\quad 5 = c_2 1^2 + c_1 1 + c_0 \rightarrow 5 = c_2 + c_1 + c_0$

$(-1,3)$ $\quad 3 = c_2 -1^2 + c_1 -1 + c_0 \rightarrow 3 = c_2 - c_1 + c_0$

d. $c_1 = 7 - c_2 \rightarrow c_1 = 7 - 6 = 1$

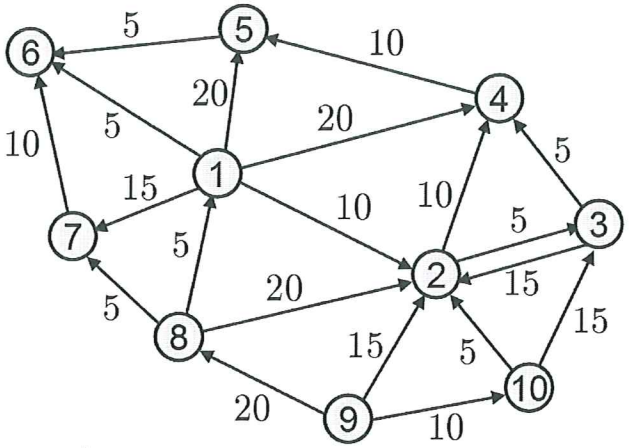
$c_2 = 5 + c_1 \rightarrow c_2 = 5 + (7 - c_2)$

$2c_2 = 12$

$c_2 = 6$

$c_2 = 6, \quad c_1 = 1, \quad c_0 = -2$

5) (15 Points) Find the shortest path for "9" to all other nodes using Dijkstra's Algorithms. Give the table



	9	8	2	10	7	1	4	3	6	5
	0	∞	∞	∞	∞	∞	∞	∞	∞	∞
9		(20)	(15)	(10)						
8			X		(25)	(25)				
2							(25)	(20)		
10			X					X		
7									35	
1			X		X		X	(30)	45	
4										(35)
3							X			
6	none									
5									X	

8 = 9, 8

2 = 9, 2

10 = 9, 10

7 = 9, 8, 7

1 = 9, 8, 1

4 = 9, 2, 4

3 = 9, 2, 3

6 = 9, 8, 1, 6

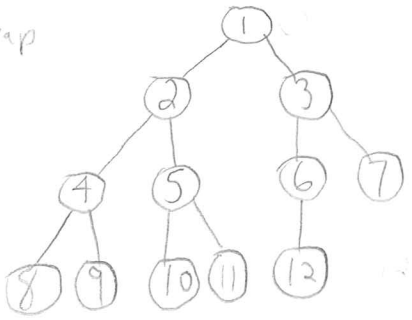
5 = 9, 2, 4, 5

6) (10 Points) We covered binary heaps in class and gave the worst-case running time. There are other types of heaps; two other common heaps are Binomial Heaps and Fibonacci Heaps. **Give an example of a Binary Heap, a Binomial Heap and a Fibonacci heap with 12 nodes**. Below are the answers we found in class for binary heap. Find the amortized time (average over n operation) for Binomial and Fibonacci heaps (cite the textbook or online reference).

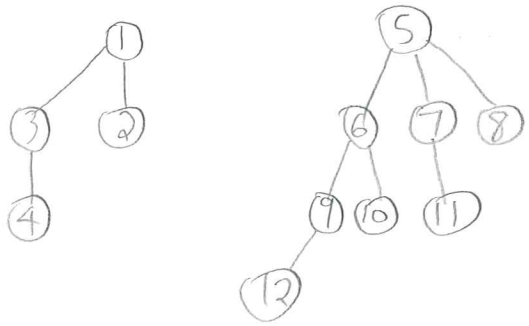
Proc	*Binary Heap (worst-case)*	*Binomial (worst-case)*	*Binomial (amortized)*	*Fibonacci (amortized)*
MakeHeap	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$
Insert	$\Theta(\lg n)$	$O(\lg n)$	$O(\lg n)$	$\emptyset(1)$
Minimum	$\Theta(1)$	$O(1)$	$\Theta(1)$	$\Theta(1)$
Extract-Min	$\Theta(\lg n)$	$O(\lg n)$	$O(\lg n)$	$O(\lg n)$
Union (Merge)	$\Theta(n)$	$O(\lg n)$	$O(\lg n)$	$\emptyset(1)$
Decrease Key	$\Theta(\lg n)$	$\Theta(\lg n)$	$O(\lg n)$	$\emptyset(1)$
Delete	$\Theta(\lg n)$	$\Theta(\lg n)$	$O(\lg n)$	$O(\lg n)$
			citation: www.cse.ust.hk/~ngolin/COMP572/Notes/Heaps.ps	citation: Wikipedia - Fibonacci heap entry

6.

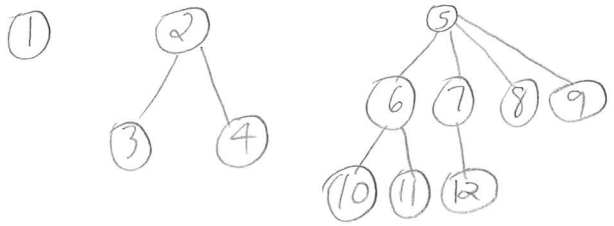
Binary Heap



Binomial Heap



Fibonacci Heap



7) (15 Points) Currently what is the fastest for the following. Give the name and running time.
a) Matrix Multiplication Algorithm?

a) Integer Multiplication Algorithm?

b) Polynomial Multiplication Algorithm?

c) Minimal Spanning Tree Algorithm? What data structure if given?

d) Shortest Path? What data structure if given?

e) Factoring? (Just name, not running time as it is a NP-Complete Problem)

a. Strassen's algorithm - $\approx O(n^{2.807})$ (Wikipedia)

b. Schönhage - Strassen algorithm - $O(n \log n \log\log n)$ (Wikipedia)

c. Kruskal's algorithm - $O(n \lg n)$ with simple data structures - (Kruskal's algorithm Wiki)

d. Dijkstra's algorithm - $O(n \lg n)$ with a fibonacci heap (Wikipedia)

e. Dixon's algorithm (Integer factorization wikipedia)