# Test 1 Practice

Name____________________

1 a) Order the $\Theta$ classes from lowest to highest of the following

$F_{10}, F_1, F_4, F_3, F_8, F_7, F_6, F_2, F_5, F_9$

2 $F_1(n) = \lg n$
8 $F_2(n) = \lg n * 3^n$
4 $F_3(n) = n$
3 $F_4(n) = \sqrt{n}$
9 $F_5(n) = n * 3^n$
7 $F_6(n) = 3^n$
6 $F_7(n) = n^{2.5}$
5 $F_8(n) = n^2(\lg n)$
10 $F_9(n) = n!$
1 $F_{10}(n) = 7$

b) Give a function in each of the following; where possible give a different function then *n*

$n >$ $\geq$ • $\Omega(n)$ - the set of all functions that grow $\leq$ the rate of n
$o <$ $\leq$ • $O(n)$
$=$ • $\Theta(n)$

2) a) Given a set of n points, write the code to find the closest pair of points. $(x,y)$

two for loops

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

for each point
  for each point

b) What is the running time of your code in part a.

3) a) Suppose we have pow (n,i) calculates n^i. Show the following code runs in O( lg(n!)) time, which is O(n(lg n))

```
Double p(double x) //input is x
{
Temp=a0
For i=1 to n
  Temp += ai pow(x,i)
}
```

$$p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_{n-1} x^{n-1} + a_n x^n$$

temp=1 b) Write code to solve that find p(x) in O(n) time.

for( i = 0 → n)

temp = $x^i$

sum += $a_i \cdot$ temp

temp *= x

4)For T(n) = 8T(n/2)+n^k    $a = 8,\ b = 2,\ f(n) = n^k$    $T(n) = \Theta(n^3)$

a) For what values of k can we use Case 1 of Masters Method? What would the running time for that value of k?    $k < 3$    $T(n) = \Theta(n^3)$

b) For what values of k can we use Case 2 of Masters Method? What would the running time for that value of k?    $K = 3$    $T(n) = \Theta(n^3 \lg n)$

c) For what values of k can we use Case 3 of Masters Method? What would the running time for that value of k?    $k > 3$    $T(n) = \Theta(n^k)$ where $af(\frac{n}{b}) \le cf(n)$

$8(\frac{n}{2})^k < cn^k$

$8/2^k < c$

5)
Consider the following pseudocode, where n is a nonnegative integer.

```
x = 0;
i = 0;

while i < n do
        x = x + 2i;
        i = i + 1;
end
```

Which of the following is a loop invariant for the while statement?

A) What does the code above do?
B) Find the complexity of the code based on n.

6)
a) Write a divide and conquer algorithm that finds the maximum item in a given list by diving the list into half and conquering each half iteratively. Assume the list is not sorted in any order.
b) Give T(n) for a) and solve with Masters Method

max( list, first, last)
{
if(last < first)
return

Max ( max(list, first, $\lfloor \frac{first + last}{2} \rfloor$)
max((list, $\frac{first + last}{2}$, last)

$2T(n/2) + 3$

7) Consider finding the minimum item of a list of unsorted numbers $a_1, a_2, \ldots a_n$.

a) What is the worst case running time? $O(n)$

b) What is the best case running time? $O(1)$

8) The following functions are arranged in order from lowest to highest order.

| | | | |
|---|---|---|---|
| 1024 | lg( n ) | n | n lg( n ) |
| $n^2$ | $3^n$ | n! | |

9) Insert the following functions in their correct position in the above list.

after n
$n \lg^2(n)$

after lg n
$\lg(n^2)$

after n
$\lg(8^n)$
$\lg(2^{3n})$
$3n$

$\sqrt[3]{n^7} = n^{2.3}$

$\sqrt{\lg(n)}$
before lg n

10) Determine , using either of the two methods demonstrated in class, whether $f(n) \in O(g(n))$ where $f(n) = 13 + 4n^2 + 15n^3$ and $g(n) = n^4$.

$\lim_{n \to \infty} \frac{13}{n^4} + \frac{4n^2}{n^4} + \frac{15n^3}{n^4}$

$32\, g(n) > f(n),\ n > 1$

$32n^4 = 15n^4 + 4n^4 + 13n^4,\ n \geq 1$

11) Given the following recurrence

T(n) = 2T(n-1) + 1

T(1) = 6

Find the explicit form of the recurrence **by the substitution method.**

$T(n) = 2T(n-1) + 1$

$T(n-1) = 2(T(n-2) + 1)$

$T(n) = 2(2T(n-2) + 1) + 1$

$T(n) = 2^2 T(n-2) + 2^1 + 1$

$2^3 T(n-3) + 2^2 + 2^1 + 1$

$2^k T(n-k) + 2^{k-1} + 2^1 + 1$

$T(1) = T(n - (n-1))$

$2^{n-1} \cdot 6 + 2^{n-1} - 1$

$7 \cdot 2^{n-1}$

$2^{n-1} T(1) + 2^{n-2} + 2^{n-1} + \ldots + 2^1 + 1$

$\sum_{i=0}^{n} 2^i = 2^{n+1}$