Proofs

* Definitions
- even - $2k$ for some $k \in \mathbb{Z}$
- odd - $2k+1$ for some $k \in \mathbb{Z}$
- rational - $\frac{a}{b}$, $a + b \in \mathbb{Z}$, $b \neq 0$
- irrational - not rational

* Proofs

ex - prove sum of two odd $\mathbb{Z}$ is even
- let m & n be odd
- so $n = 2k+1$ and $m = 2j+1$
- thus $m+n = 2k+1+2j+1 = 2(k+j+1)$, is even $\boxed{/}$

* Indirect Proofs using Contrapositive
- $p \to q \equiv \sim q \to \sim p$

direct ex - if n^2 is even, then n is even

suppose n^2 is even

so $n^2 = 2k$, $n = \sqrt{2k}$ stuck!

indirect ex - if n is odd, then n^2 is odd

since n is odd, $n = 2k+1$ for some $k \in \mathbb{Z}$

so $n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$ is odd

* Contradiction
- $p \to q$, assume p is true, assume q is false, $(p \to \sim q)$

ex - suppose n^2 is even and n is odd

well $n = 2k+1$, so $n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$, so odd

but we know n^2 is even, a contradiction $\blacksquare$

* argument

- if $p \to q$ and $q \to r$, then $p \to r$
- if p is true and $p \to q$, then q = true
- truth table ↗

$(p \wedge (p \to q)) \to q$

p	q	$p \to q$	$p \wedge (p \to q)$	Q
1	1	T	T	T
1	0	F	F	T
0	1	T	F	T
0	0	T	F	T

- a tautology is a valid argument, if it is not, it is not valid
-

p	q	$p \to q$	$p \to q \wedge q$	$((p \to q) \wedge q) \to p$
T	T	T	T	
T	F	F	F	
F	T	T	T	
F	F	T	F	

not valid

- p if and only if q, prove if $p \to q$ and if $q \to p$
- sufficient = $p \to q$
- necessary = $q \to p$