Brandon Buchanan

Homework 5

1. base $2(1) = 1(1+1)$, is true. $\boxtimes$

assume P(k) $2+4+6+\ldots 2k = k(k+1)$

P(k+1) $2+4+6+\ldots 2k + 2(k+1) = k+1(k+2)$

$(k+1)(k+2) = k(k+1) + 2(k+1)$

$k^2 + k + 2k + 2$

$k^2 + 3k + 2 = (k+1)(k+2)$, is true. $\boxtimes$

2. base $(2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$

$(2(1)-1)^2 = \frac{1(2(1)+1)(2(1)-1)}{3}$

$1 = \frac{1(3)(1)}{3}$ is true. $\boxtimes$

assume P(k) $1^2 + 3^2 + \ldots (2k+1)^2 = \frac{k(2k+1)(2k-1)}{3}$

P(k+1) $1^2 + 3^2 + \ldots (2k+1)^2 + (2k+2)^2 = \frac{(k+1)(2k+2)2}{3}$

$\frac{k(2k+1)(2k-1)}{3} + \frac{3(2k+2)^2}{3} = \frac{(k+1)(2k+2)2}{3}$

$k(2k+1)(2k-1) + 3(2k+2)^2 = (k+1)(2k+2)2$

$k(4k^2-1) + 3(4k^2+8k+4) =$

$4k^3 + 5k + 6 + 12k^2 + 24k + 12 =$

3. base $1 + 2 = 2^2 - 1$, is true. $\boxtimes$

assume P(k) $1 + 2 + 2^2 + \ldots 2^k = 2^{k+1} - 1$

P(k+1) $1 + 2 + 2^2 + \ldots 2^k + 2^{k+1} = 2^{k+2} - 1$

$2^{k+1} - 1 + 2^{k+1} = 2^{k+2} - 1$

$2^{k+2} - 1 = 2^{k+2} - 1$, is true. $\boxtimes$

4 base $5 = \frac{5(1+1)}{2}$ is true ☒

assume $P(k)$ $5 + 10 + \dots, 5k = \frac{5k(k+1)}{2}$

$P(k+1)$ $5 + 10 + \dots, 5k + 5(k+1) = \frac{5(k+1)(k+2)}{2}$

$\frac{5k(k+1)}{2} + \frac{10(k+1)}{2} = \frac{5(k+1)(k+2)}{2}$

$5k(k+1) + 10(k+1) = 5(k+1)(k+2)$

$5k^2 + 5k + 10k + 10 =$

$5k^2 + 15k + 10 =$

$5(k^2 + 3k + 2) =$

$5(k+1)(k+2) =$, is true ☒

5. base $1^2 = \frac{1(1+1)(2+1)}{6}$ is true ☒

assume $P(k)$ $1 + 2^2 + k^2 = \frac{k(k+1)(2k+1)}{6}$

$P(k+1)$ $1^2 + 2^2 + \dots k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$

$\frac{k(k+1)(2k+1)}{6} + \frac{6(k+1)^2}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$

$\frac{k(k+1)(2k+1) + 6(k+1)^2}{6} =$

$\frac{k+1(k(2k+1) + 6(k+1))}{6} =$

$\frac{k+1(2k^2 + 7k + 6)}{6} =$

$\frac{(k+1)(k+2)(2k+3)}{6} =$, is true ☒

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6 base $1 = \frac{a^1 - 1}{a-1}$, is true ☒

assume $P(k)$ $1 + a + a^2 + \dots a^{k-1} = \frac{a^{k-1}}{a-1}$

$P(k+1)$ $1 + a + a^2 + \dots + a^{k-1} + a^k = \frac{a^k}{a-1}$

$\frac{a^{k-1}}{a-1} + a^k = \frac{a^k}{a-1}$

$\frac{a^{k-1}}{a-1} + \frac{a^k(a-1)}{a-1} = \frac{a^k}{a-1}$

$a^{k-1} + a^k(a-1) = a^k$

$a^{k-1} + a^{k+1} - a^k = a^k$

$2a^k - a^k = a^k$, is true ☒

7. base $a = \frac{a(1-r)}{1-r}$, is true ☒

$P(k)$ $a + ar + ar^2 + \dots ar^{k-1} = \frac{a(1-r^k)}{1-r}$

$P(k+1)$ $a + ar + \dots ar^{k-1} + ar^k = \frac{a(1-r^{k+1})}{1-r}$

$\frac{a(1-r^k)}{1-r} + ar^k = \frac{a(1-r^{k+1})}{1-r}$

$a(1-r^k) + ar^k(1-r) = a(1-r^{k+1})$

$a - ar^k + ar^k - ar^{k+1} =$

$a - ar^{k+1} =$

$a(1-r^{k+1}) =$, is true ☒

8. base $1^3 + 1^3 = \frac{1(2^2)+4}{4}$

$2 = \frac{4+4}{4}$, is true ☒

$P(k)$ $1^3 + 2^3 + \dots k^3 = \frac{k^2(k+1)^2 + 4}{4}$

$P(k+1)$ $1^3 + 2^3 + \dots k^3 + (k+1)^3 = \frac{(k+1)^2(k+2)^2 + 4}{4}$

$k^2(k+1)^2 + 4 + (k+1)^3 = (k+1)^2(k+2)^2 + 4$

$\left((k+1)^2\left(k^2 + (k+1)\right)\right) + 4 =$

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HW 5

9. base a | $4(1)-3 = (2n+1)(n-1)$
$1 = 3 \cdot 0$, is false. ☒

b | no, because the base step is false.

10. base | $1+2^0 < 3^0$, is true. ☒

$P(k)$ | $1+2^k < 3^k$

$P(k+1)$ | $1+2^{k+1} < 3^{k+1}$

$3(1+2^k) < 3(3^k)$

notice $3+3\cdot 2^k > 1+3\cdot 2^k$

also $1+2\cdot 2^k < 1+3\cdot 2^k$

so $1+2^{k+1} < 1+3\cdot 2^k < 3+3\cdot 2^k < 3^{k+1}$

11. base | $2 < 2^2$, is true. ☒

$P(k)$ | $k < 2^k$

$P(k+1)$ | $k+1 < 2^{k+1}$

$k+1 < 2^k+1$

$2^k+1 < 2^{k+1}$

thus $k+1 < 2^k+1 < 2^{k+1}$, is true ☒

12. base | $1 < \frac{3^2}{8}$ is true. ☒

$P(k)$ | $1+2+\dots k < \frac{(2k+1)^2}{8}$

$P(k+1)$ | $1+2+\dots k+k+1 < \frac{(2k+2)^2}{8}$

$k+1 + \frac{(2k+1)^2}{8} > \frac{(2k+2)^2}{8}$

$\frac{8(k+1)}{8} + \frac{(2k+1)^2}{8} > \frac{(2k+2)^2}{8}$

$8(k+1) + (2k+1)^2 > (2k+2)^2$

$8k+8+4k^2+4k+1 > 4k^2+8k+4$

$4k^2+12k+9 > 4k^2+8k+9$, is true. ☒

13. $n = 5$ is the least $1 + 5^2 < 2^5$

$P(k)$ $1 + k^2 < 2^k$

$P(k+1)$ $1 + (k+1)^2 < 2^{k+1}$

$k^2 + 2k + 2$

14. $n = 4$ is the least $10(4) < 3^4$

$P(k)$ $10k < 3^k$

$P(k+1)$ $10(k+1) < 3^{k+1}$

$3(10(k+1)) < 3(3^k)$

15. base A has 1 e., so it has 2 elements in $P(A)$

$P(k)$ A has k e., so it has 2^k elements in $P(A)$

$P(k+1)$ $k+1$ 2^{k+1}

$k+1 = {}_kC_{k-0} + {}_kC_{k-1} + \ldots {}_kC_{k-n}$, where the last $n = k$