1

Brandon Buchanan
Math CS
2.22.07

Homework 4

2.2

2\. a $\sim p \leftrightarrow q$

b $q \rightarrow p \wedge r$

c $\sim r \rightarrow p \wedge \sim q$

d $\sim p \wedge q \rightarrow r$

3\. a If I am not the Queen of England, then 2 + 2 = 4

b. If I walk to work, then I am not the President of the US.

c If I did not take the train to work, then I am late

d. If I go to the store, then I have time and am not too tired

e If I buy a car and a house, then I have enough money

4\. a If I am the Queen of England, then 2 + 2 ≠ 4.

b If I do not walk to work, then I am the President of the US.

c. If I did take the train to work, then I am not late.

d. If I don't go to the store, then I don't have time or am too tired

e. If I don't buy a car or a house, then I don't have enough money

8\. a $p \vee q$

b $r \rightarrow \vee s$

c $\sim(s \vee p)$

10 a

p	~p	Q
0	1	0
0	1	0
1	0	0
1	0	0

absurdity

b.

contingency

q	p	~q	~q ∧ p	q ∨ ~q ∧ p
0	0	1	0	0
0	1	1	1	1
1	0	0	0	1
1	1	0	0	1

11\. a

p	q	q → p	p → (q → p)
0	0	1	0
0	1	0	1
1	0	1	1
1	1	1	1

contingency

b

q → (q → p)
1
0
1
1

contingency

12 a

q	p	$q \wedge p$	$q \wedge \sim p$	Q
0	0	0	0	0
0	1	0	0	0
1	0	0	1	1
1	1	1	0	1

contingency

b

p	q	$p \wedge q$	$p \wedge q \to p$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	1

tautology

c

p	q	$q \wedge p$	$p \to (q \wedge p)$
0	0	0	1
0	1	0	1
1	0	0	0
1	1	1	1

contingency

17. a false
b true
c false $\wedge$ true = false
d true

18. a false $\to$ (true $\to$ (true $\to$ (true $\vee$ false))) = true
b true $\to$ (false $\to$ true) = true
c true $\wedge$ false = false
d true

21. a Jack did eat fat, and he ate broccoli
b Mary did not lose her lamb and the wolf did not eat the lamb.
c If Tom did not steal a pie and did not run away, then the three pigs did have suppor

22. a I
b IV

Homework 4

2.3

1. $((p \to q) \wedge \sim q) \to \sim p$

p	q	~p	~q	p→q	(p→q) ∧ ~q	Q
0	0	1	1	1	1	1
0	1	1	0	1	0	1
1	0	0	1	0	0	1
1	1	0	0	1	0	1

valid

2. $((p \to q) \wedge q) \to p$

p	q	p→q	(p→q) ∧ q	Q
0	0	1	0	1
0	1	1	1	0
1	0	0	0	1
1	1	1	1	1

not valid

3. $((p \to q) \wedge \sim p) \to \sim q$

p	q	~p	~q	p→q	(p→q) ∧ ~p	Q
0	0	1	1	1	1	1
0	1	1	0	1	1	0
1	0	0	1	0	0	1
1	1	0	0	1	0	1

not valid

4. $((p \to q) \wedge p) \to q$

(p→q) ∧ p	Q
0	1
0	1
0	1
1	1

valid

5. $((p \vee q) \wedge q) \to p$

p	q	p ∨ q	(p ∨ q) ∧ q	Q
0	0	0	0	1
0	1	1	1	0
1	0	1	0	1
1	1	1	1	1

not valid

6. $((p \vee q) \wedge \sim q) \to p$

p ∨ q	(p ∨ q) ∧ ~q	Q
0	0	1
1	0	1
1	1	1
1	0	1

valid

7. $((p \wedge q \to r) \wedge (r \to s)) \to (\sim s \to (\sim p \vee \sim q))$ valid

p	q	r	s	$p \wedge q$	$p \wedge q \to r$	$r \to s$	$Q \wedge R$	$\sim p \vee \sim q$	$\sim s \to (\sim p \vee \sim q)$	S
0	0	0	0	0	1	1	1	1	1	1
0	0	0	1	0	1	1	1	1	1	1
0	0	1	0	0	1	0	0	1	1	1
0	0	1	1	0	1	1	1	1	1	1
0	1	0	0	0	1	1	1	1	1	1
0	1	0	1	0	1	1	1	1	1	1
0	1	1	0	0	1	0	0	1	1	1
0	1	1	1	0	1	1	1	1	1	1
1	0	0	0	0	1	1	1	1	1	1
1	0	0	1	0	1	1	1	1	1	1
1	0	1	0	0	1	0	0	1	1	1
1	0	1	1	0	1	1	1	1	1	1
1	1	0	0	1	0	1	0	0	0	1
1	1	0	1	1	0	1	0	0	1	1
1	1	1	0	1	1	0	0	0	0	1
1	1	1	1	1	1	1	1	0	1	1

8. $((g \to p) \wedge (\sim s \to \sim p) \wedge (s \to \sim v)) \to (v \to \sim g)$
$(g \to p) \wedge (p \to s) \wedge (s \to \sim v) \to (g \to \sim v)$

valid using inference

9. $((p \to \sim b) \wedge (h \to p) \wedge h) \to b$
$(h \to p \to \sim b) \to b$ ∴ not valid using inference

10. a $((p \vee q) \wedge \sim q) \to p$

valid using indirect method

b $((p \to q) \wedge \sim p) \to \sim q$

p	q	$p \to q$	$(p \to q) \wedge \sim p$	Q
0	0	1	1	1
0	1	1	1	0
1	0	0	0	1
1	1	1	0	1

not valid

11. a $((p \vee q) \wedge \sim q) \to p$

b. $((p \to q) \wedge \sim p) \to \sim q$

Homework 4

12 a $((p \to q) \wedge (q \to r) \wedge (\sim q \wedge r)) \to p$

- valid using inference
- since $p \to q \to r$, LHS always evaluates to false, and $f \to t$ or $f \to f$ is true

b $(\sim(p \to q) \wedge p) \to \sim q$

$(\sim q \to \sim p) \wedge p \to \sim q$

$\sim q$	$\sim p$	$\sim q \to \sim p$	$(\sim q \to \sim p) \wedge p$	q
0	0	1	1	0
0	1	1	0	1
1	0	0	0	1
1	1	1	0	1

not valid

14. - Let m & n be even

so $m = 2k$ & $n = 2j$

thus $m+n = 2k + 2j = 2(k+j)$, which is even. ☒

15. - Let m & n be odd

so $m = 2k+1$ & $n = 2j+1$

thus $m+n = 2k + 2j + 2 = 2(k+j+1)$, which is even. ☒

16. - If n is odd, then n^2 is odd

Since n is odd, $n = 2k+1$ for some $k \in \mathbb{Z}$

so $n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$, which is even ☒

- If n is even, then n^2 is even,

Since n is even, then $n = 2k$ for some $k \in \mathbb{Z}$

so $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$, which is even ☒

21 a $A \subseteq B \leftrightarrow A \cup B = B$

- Pick $x \in A \subseteq B$, show in $A \cup B$

Since $A \subseteq B$, $x \in A$ and $x \in B$

Hence $x \in A \cup B$

Because $A \subseteq B$, $A \cup B = B$

- Pick $x \in A \cup B$, show in $A \subseteq B$

Since $x \in A \cup B$, $x \in A$ + $x \in B$

Hence $x \in A \subseteq B$

Because $A \cup B = B$, then $A \subseteq B$

b $A \subseteq B \leftrightarrow A \cap B = A$

- Pick $x \in A \subseteq B$, show in $A \cap B$

Since $A \subseteq B$, $x \in A$ + $x \in B$

Hence $x \in A \cap B$

Because $A \subseteq B$, $A \cap B = A$

- Pick $x \in A \cap B$, show in $A \subseteq B$

Since x $A \cap B$, $x \in A$ + $x \in B$

Hence $x \in A \subseteq B$

Because $A \cap B = A$, then $A \subseteq B$

22 1 - If k is odd, then k^3 is odd

2 - If k^3 is odd, then k is odd

1 If k is even, then k^3 is even

So $k = 2n$

Thus $k^3 = 2n^3 = 8n^3 = 2(4n^2)$, which is even ☐

2. Proved both ways because the first statement using the converse, while the second was the contradiction.

Homework 4

24. $n + n+1 + n+2 + n+3 + n+4 = 5n+10$

- Let n be an integer

So $n+(n+1)+(n+2)+(n+3)+(n+4) = 5n+10$

And $5n+10 = 5(n+2)$, which is divisible by 5. ☒

25. $3 \mid (n^3 - n)$

- So $3 \mid n(n+1)(n-1)$

Thus $n(n+1)(n-1) = 3q$