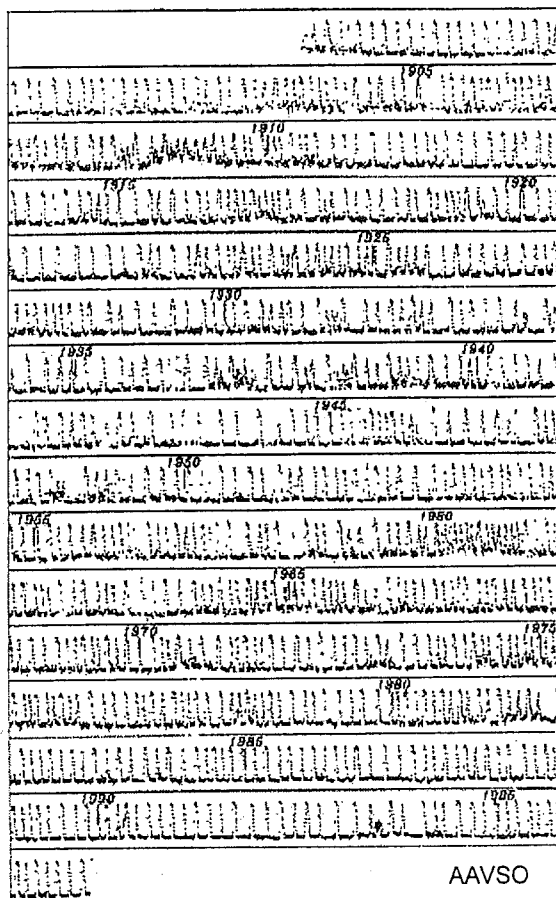


Chapter 11: Variable Stars, Light Curves, and Periodicity

Introduction

Variable stars are, quite simply, stars that change brightness. More than 30,000 are known and catalogued, and many thousands more are suspected variables. Their light variations are a vital clue to their nature, but professional astronomers do not have the time or the telescopes to monitor the brightness of thousands of variables night after night; their efforts are directed toward understanding specific aspects of the stars using such sophisticated instruments as spectrographs, photometers, radio telescopes, and satellites. Such instruments enable us to probe many different regions of the spectrum, such as X-rays, ultraviolet, infrared, and radio wavelengths. Yet it is crucial to measure the stars in ordinary visual light (the *optical* region of the spectrum) as well. The data obtained with special instruments and in shorter or longer wavelengths of light are compared with optical measurements, which serve as a



100-year light curve of AAVSO observations
of the variable star SS Cygni.

benchmark. By observing variable stars, a serious observer (such as an amateur astronomer or student) can make a significant contribution to astronomy. Also, to understand and create theories about why and how stars vary, astronomers need to know the long-term history of the stars; hence it is essential that we have long-term observations. To date, the vast majority of long-term data has been provided by amateur observers.

Observations of variable stars are plotted on a graph called a *light curve* as the apparent brightness (magnitude) versus time, usually in Julian Date (JD). The light curve is the single most important graph in variable star astronomy. The light curve allows astronomers to unlock some of the secrets of stars and decode the messages hidden within the starlight. Information about the periodic behavior, the orbital period of eclipsing binaries, or the regularity (or lack) of stellar eruptions can be directly determined from the light curve. More detailed analysis of the light curve allows astronomers to calculate

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such information as the masses or sizes of stars. Several years' worth of observational data can reveal the changing period of a star, which is a signal of a change in the structure of the star. In the same way that a histogram is a useful tool to inspect the precision of a set of measurements, we can visualize the nature of a star's variation more easily by plotting a light curve of apparent magnitude versus time.

Light curves show that many variable stars are periodic. Periodic phenomena repeat in a regular way that is predictable. However, the line between periodic phenomena and non-periodic phenomena is not always a sharp one. Sometimes a process may seem to be periodic from a large-scale perspective, while investigation on a fine scale reveals non-periodic variations. Consider the change in apparent altitude of the Sun. If you measure the highest apparent altitude of the Sun each day for a week, you will not notice much change. The Sun seems to reach the same maximum altitude each day. Over a period of a month, however, changes will become obvious. But if we take measurements each day from December 31st until the summer solstice around June 21st, the apparent daily altitude of the Sun gets higher and higher. This is still non-periodic behavior. If we take our measurements for an entire year, from December 31st to December 31st, we will see a definite pattern. When we examine the change in the apparent altitude of the Sun over several years, we see that in addition to monthly variations, it follows a regular pattern with a period of one year.

Many variable stars do exhibit simple periodic behavior. Some semiregular variables have time intervals of periodic behavior in between non-periodic intervals. Some have two separate periodic cycles superimposed over each other, affecting the overall result. The only way to determine the periodicity is by plotting and analyzing light curves. The emerging patterns will then tell their story.

There are four main classes of variable stars: *pulsating* and *eruptive* variables whose variability is *intrinsic*—due to physical changes in the star or stellar system; and *eclipsing binary* and *rotating* stars whose variability is *extrinsic*—due to an eclipse of one star by another or the effect of stellar rotation. Below is a brief description of several different types of variable stars with their corresponding light curves. There is also more detailed information about some of the variables mentioned below throughout the *Hands-On Astrophysics* manual. A general discussion of variables is the focus of the Space Talk in Chapter 5 (pp. 87–88); the Space Talk in Chapter 7 (pp. 118–121) is about Cepheids and the distance scale; the Space Talk in Chapter 12 (pp. 225–228) is about omicron Ceti (Mira); Poster Page 12.1 in Chapter 12 features SS Cygni; and Space Talk on pages 248–250 in Chapter 13 is about beta Persei (Algol). In addition, a summary of the different types of variable stars is included in the Appendix, and a discussion of variable stars is presented in the accompanying HOA video entitled *Variable Stars*.

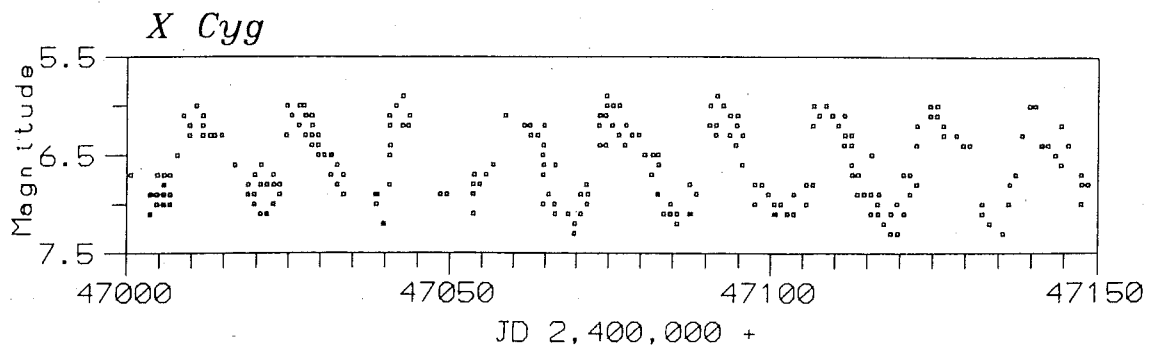
I. Intrinsic Variable Stars

A. Pulsating Variable Stars

Pulsating variables change brightness because they change their size and/or shape; the whole star is actually “vibrating.” Most of them simply expand and contract repeatedly, swelling and shrinking in a continuing cycle of size changes; this is known as the *fundamental mode of pulsation*. Others change not only their size, but the internal arrangement of material within the star as well. Still others change shape, alternately flattening and elongating, or showing even more complex shape changes. And to make things yet more complicated, some stars vibrate in more than one way (more than one *mode*) at the same time. Like most vibrating systems, pulsating variables repeat their changes; they tend to be *periodic*. (See the Space Talk in Chapter 12, pp. 225–228.)

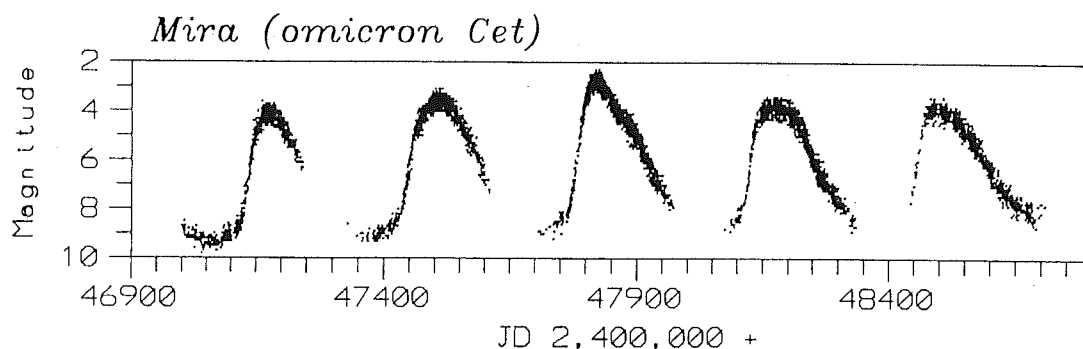
One important class of pulsating variables is the *Cepheid variables*. These large yellow stars pulsate with periods from 1 to 70 days, with an amplitude of light variation up to 2 magnitudes. They are intrinsically very bright (high luminosity). The greater the absolute magnitude (luminosity) of a Cepheid, the longer its period. In fact, there is a strict relationship between a Cepheid’s period and its luminosity, called the *period-luminosity relationship*. We can determine the period of a Cepheid from its light curve; then, using the period of the Cepheid, we can apply the period-luminosity relationship to compute the Cepheid’s luminosity. Knowing its magnitude (how bright it appears to observers on Earth) and its luminosity (how bright it actually is) allows us to compute its distance from us, thereby enabling us to use Cepheid variables as distance markers. For example, just as astronomers in the first half of this century used Cepheid variables within the Milky Way itself to measure distances within our own galaxy, we can measure the distance to any given galaxy by computing the distance to its Cepheid variables. (See also the discussion in the Space Talk in Chapter 7, pp. 118–121.)

The light curve for the Cepheid variable X Cygni is shown below; each dot represents one observation. (All observations of X Cyg, and all subsequent light curves come from the AAVSO International Database, unless otherwise noted.)

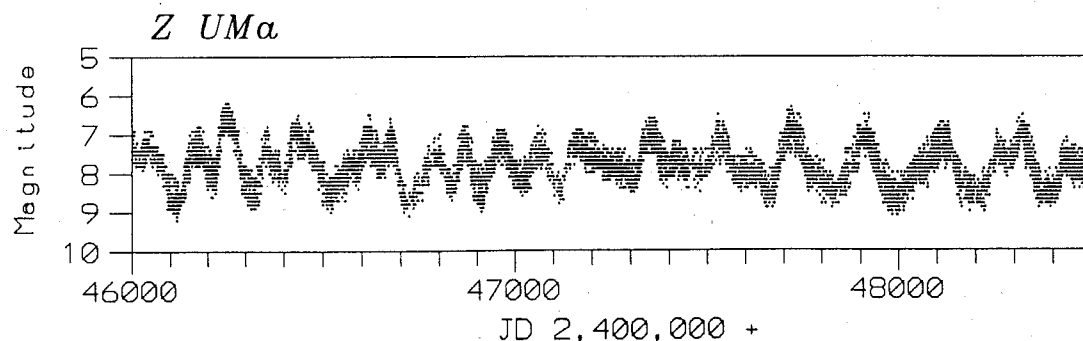


Chapter 11: Variable Stars, Light Curves, and Periodicity

One of the largest groups of pulsating variables is the *long-period variables*, or *LPVs*. They are further divided into two major subclasses, the *Mira-type* and the *semiregular* variables. The visual light curves of Mira-type variables show well-defined periods ranging from 80 to nearly 1000 days, with amplitudes of 2.5 magnitudes or more. Mira-type variables are red giant stars, often of enormous size. Many of them are slowly ejecting a steady stream of matter into the surrounding space; this mass loss can have very dramatic consequences for their future evolution. This class of variable is named after the star *Mira* (also known as omicron Ceti), the first such star discovered (see the Space Talk in Chapter 12, pp. 225–228); its light curve is shown below.

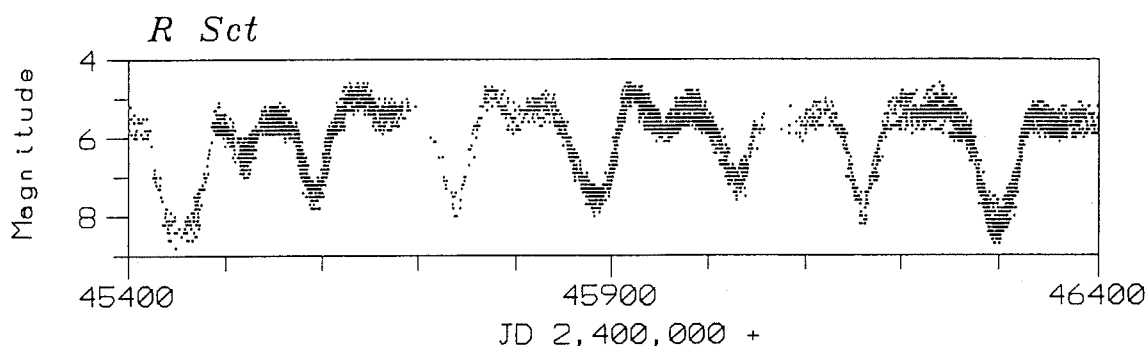


Yet another group of pulsating stars is the *semiregular variables*. These giants and supergiants show appreciable periodicity accompanied by intervals of irregular light variation. The periods range from 30 to 1000 days, with light amplitudes of not more than one to two magnitudes. An example is Z Ursae Majoris.



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RV Tauri stars are also variables that pulsate. These yellow supergiants have a characteristic light variation with alternating deep and shallow minima. Their periods, defined as the interval between two deep minima, range from 30 to 150 days. The light amplitude may vary as much as four magnitudes. Many of these variables show long-term cyclic variations in brightness from hundreds to thousands of days. An example of an RV Tauri variable star light curve is that of R Scuti.

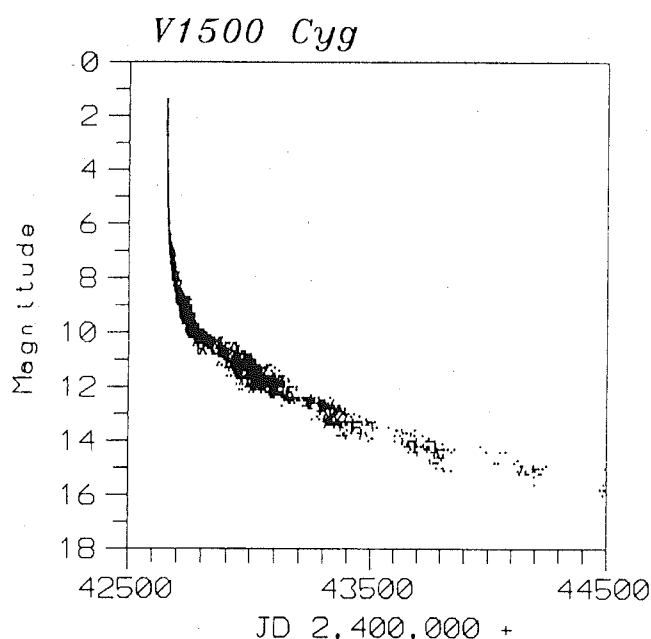


B. Eruptive (Cataclysmic) Variable Stars

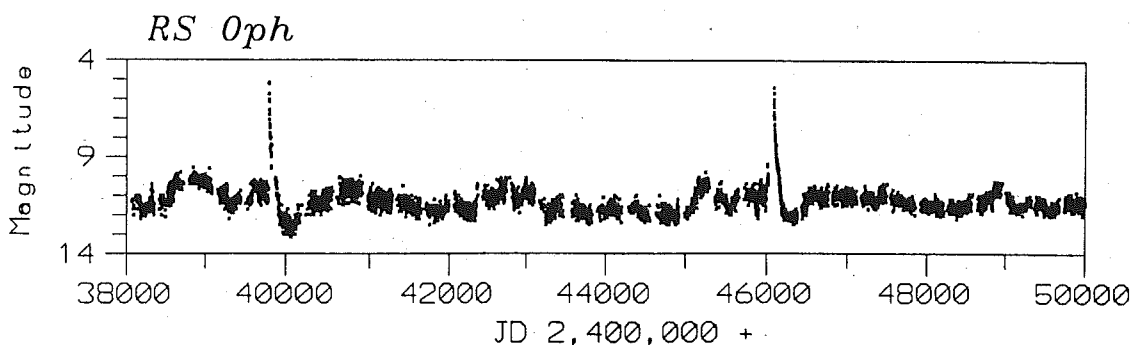
Another major category of variables is the eruptive variables. These are not one-star systems, but two stars orbiting very close to each other, one of them a normal Sun-like or giant star, the other a white dwarf star. White dwarf stars are very compressed: they may pack the mass of our entire Sun into a size no bigger than the Earth. Some of the outermost material from the larger star is pulled away by the white dwarf's gravity, but this material does not fall directly onto the white dwarf. Instead, it builds up in a disk called an *accretion disk*, which orbits the white dwarf. The combination of normal or giant star, white dwarf, and accretion disk can lead to some very spectacular celestial fireworks. That is why, instead of varying smoothly like most pulsating variables, eruptive variables exhibit *outbursts* of activity, usually brightening by a large amount. The changes in their light curves are usually very unpredictable, and tend to be sudden and dramatic; that is why these stars are also called "cataclysmic variables."

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Novae explosions (eruptions) occur in close binary systems consisting of a white dwarf orbiting a larger and cooler star. A layer of hydrogen-rich material is slowly accreted from the cooler star onto the compact white dwarf. The accreted material provides the fuel for the nova explosion—a thermonuclear fusion reaction similar to the detonation of a hydrogen bomb. The system increases in brightness by 7 to 16 magnitudes in a matter of one to several hundred days. After the eruption, the light slowly fades back to its original brightness over several years or decades, as shown in the light curve of V1500 Cygni at right.



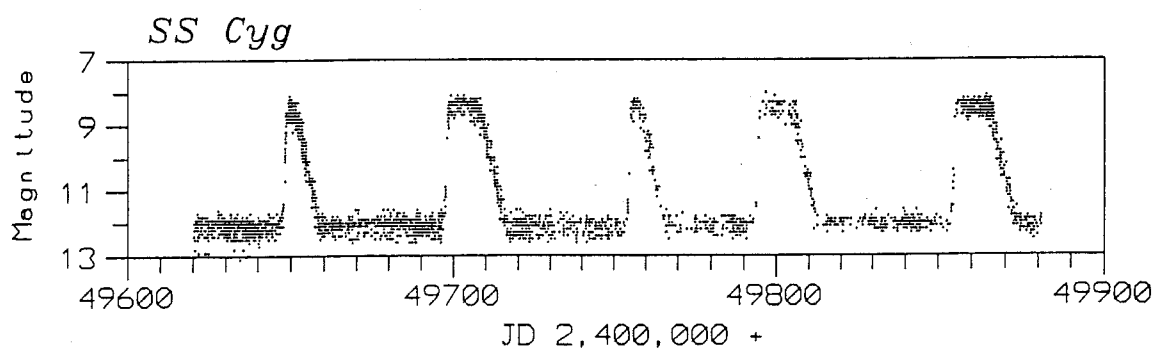
Systems which have undergone two or more nova-like eruptions during their recorded history are referred to as *recurrent novae*. Such recurrent eruptions have a slightly smaller amplitude than that which is observed in novae. An example of a recurrent nova is RS Ophiuchi.



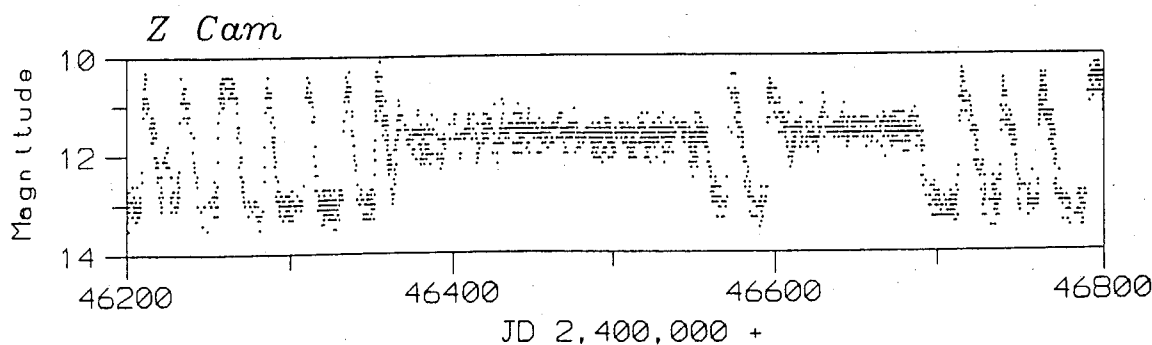
Chapter 11: Variable Stars, Light Curves, and Periodicity

Yet another class of cataclysmic variable stars is called *dwarf novae*. These are close binary systems made up of a Sun-like star, a white dwarf, and an accretion disk surrounding the white dwarf. As matter accumulates in the accretion disk, the disk becomes unstable. Eventually, matter from the unstable disk will fall onto the white dwarf, leading to an outburst. There are three sub-classes of dwarf novae.

Dwarf novae of the *U Geminorum* sub-class (also called SS Cygni sub-class) are also in close binary systems, with orbital periods on the order of a few hours. After intervals of well-defined quiescence at minimum magnitude, they brighten suddenly within 1–2 days. Depending on the star, the eruptions occur at intervals of 30 to 500 days. The light amplitude of the outbursts ranges from two to six magnitudes with a duration of 5 to 20 days. An example is SS Cygni.

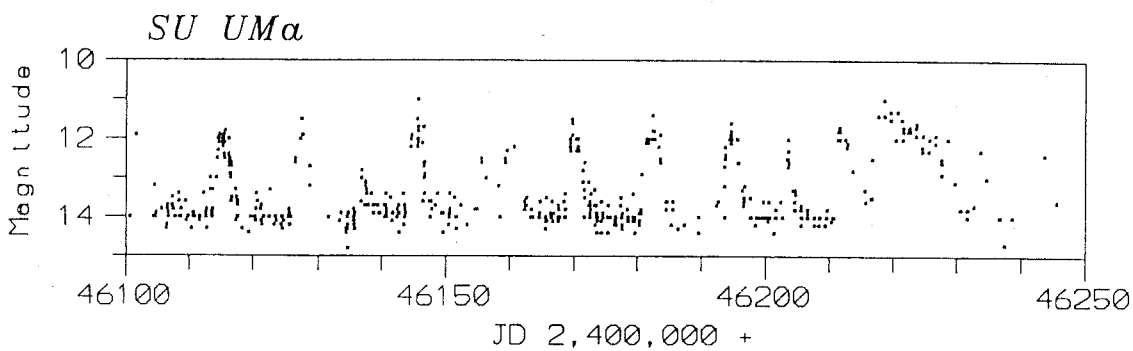


Z Camelopardalis stars, the second dwarf novae sub-class, are physically similar to *U Geminorum* stars. Their eruptions occur at intervals of 10 to 30 days or more; however, the quiescent level of *Z Camelopardalis* stars is not well-defined. In addition, the cyclic variations are interrupted by intervals of constant brightness called *standstills* (also called *stillstands*). The standstills last the equivalent of several cycles (sometimes years), with the star “stuck” at a brightness approximately a third of the way from maximum to minimum. In general, at the end of a standstill, the star continues to fade to its quiescent level, as in the light curve of the prototype star *Z Camelopardalis*, below.

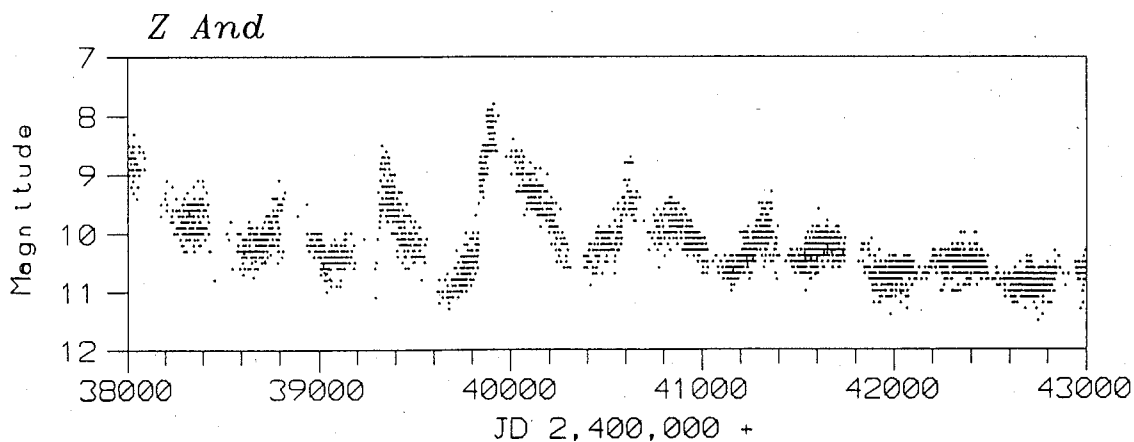


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SU Ursae Majoris stars, the third dwarf novae sub-class, are also physically similar to U Geminorum stars. They have short orbital periods of less than two hours. In addition, they have two distinct kinds of outbursts: one is short, with a duration of one to several days, faint and more frequent; the other (*superoutburst*) is long, with a duration of 10 to 20 days, bright and less frequent. During superoutbursts, small-amplitude, periodic modulations (*superhumps*) appear with periods two to three percent longer than the orbital period of the system. The light curve of *SU Ursae Majoris* appears below; can you find the superoutburst?

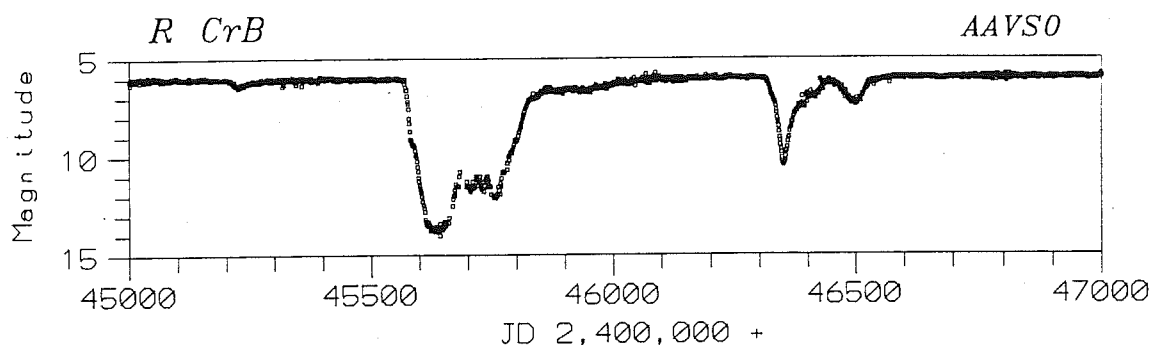


Another group of eruptive variables is the *symbiotic stars*, which display semiperiodic nova-like outbursts of up to four magnitudes. These are close binary systems with one component a red giant and the other a hot blue star. They are embedded in *nebulae*, a region of gas and dust. An example is *Z Andromedae*.



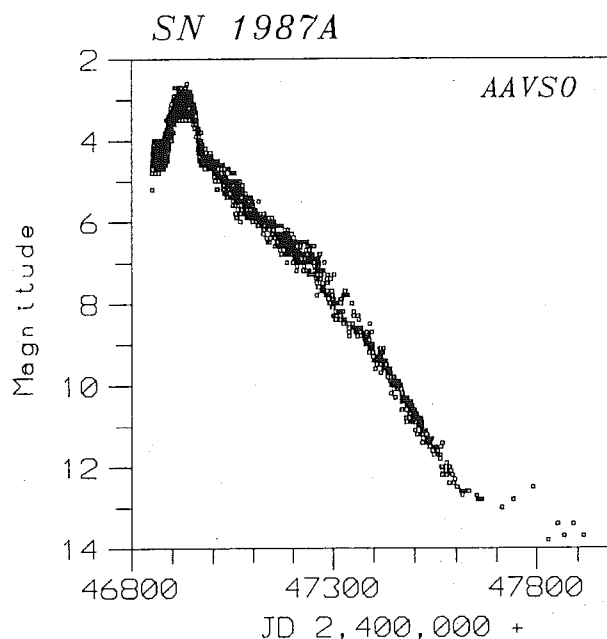
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R Coronae Borealis variables are high-luminosity stars (supergiants) which go into “outburst” not by brightening, but by fading! They spend most of their time at maximum magnitude, and at irregular intervals fade by one to nine magnitudes, slowly recovering to their maximum magnitude after a few months or years. The drop in brightness is believed to be caused by the formation of soot in the star’s atmosphere, which is abnormally rich in carbon. When the carbon veil is blown away, the star returns to maximum magnitude. The light curve (one-day means) of *R Coronae Borealis* is shown below.



Perhaps the most spectacular of all variables is the *supernova*. A supernova is the violent end of a massive star during which the star quite literally explodes, blasting most of its material out into space in seconds. As a result, it can brighten by 20 magnitudes or more; a large supernova may briefly outshine the entire rest of the galaxy. The tremendous explosion will leave behind a fragment of very dense material at its center. This may take the form of a neutron star, whose material is compressed to the density of an atomic nucleus. Or it may be so compressed that it forms a black hole, a region where the gravity is so strong that nothing, not even light, can escape.

The light curve of the spectacular Supernova 1987A, which appeared in 1987 in the Large Magellanic Cloud (a dwarf galaxy near our own Milky Way), is shown at right.



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The material blown off in a supernova explosion is rich in complex elements such as carbon, nitrogen, oxygen, and iron. Eons later, this material can condense as part of a newly-forming star system. The enrichment of our galaxy by supernova explosions has provided the necessary complex elements to form Earth-like planets, and even life. It is no exaggeration to say that we are stardust: many atoms in our bodies, including carbon, calcium, iron, and magnesium, were synthesized within the core of a star, and probably ejected by a supernova explosion.

A supernova happens, on average, about every 300 years in a galaxy. The last supernova in our own galaxy was in the year 1054, over 900 years ago. Are we overdue?

Stars vary when they are in the stages of birth, old age, or death, all of which are periods of extreme instability in the life of a star. Many pulsating variables, for example, are near the end of their lives, and running out of their main fuel, hydrogen. Thermonuclear fusion does not simply turn on at birth and quietly turn off when the hydrogen is exhausted: fuels other than hydrogen can be burned, causing a wide variety of nuclear reactions and changes to the star's structure. The changes in the fuel economy of a star can upset the delicate balance between gravity and radiation pressure which has kept the star stable for most of its lifetime. Through the variations in their brightness, these stars "talk" to us—they try to tell us their story. Thus by studying variable stars, astronomers try to put together the puzzle of how stars evolve and die.

Chapter 11: Variable Stars, Light Curves, and Periodicity

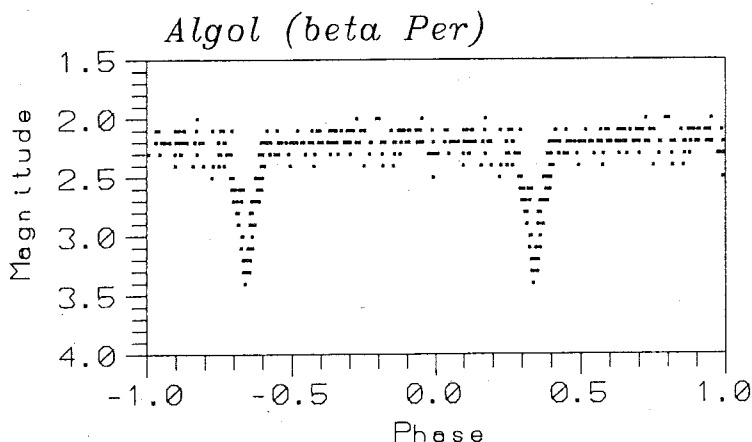
II. Extrinsic Variable Stars

A. Eclipsing Binaries

Eclipsing binaries are binary star systems whose members eclipse each other, blocking one another's light, thereby causing the system to look fainter to observers on Earth.

The light curve of an eclipsing binary depends on the sizes and brightnesses of the stars, their separation from each other, and the geometry of our view from Earth. For example, the first star may be larger, and totally eclipse the second, while the second only partially eclipses the first. The time between primary eclipses is the orbital period of the binary system. Detailed analysis of the light curve shape yields much information about the size, mass, and shape of the stars, and the shape of their orbits.

Beta Persei (Algol), whose folded light curve is shown below, is an eclipsing binary that can be seen with the unaided eye. (Folded light curves, or phase diagrams, have two or more cycles superimposed on each other. They will be examined in greater detail in Chapter 12.) See the Space Talk in Chapter 13, pages 248-250.



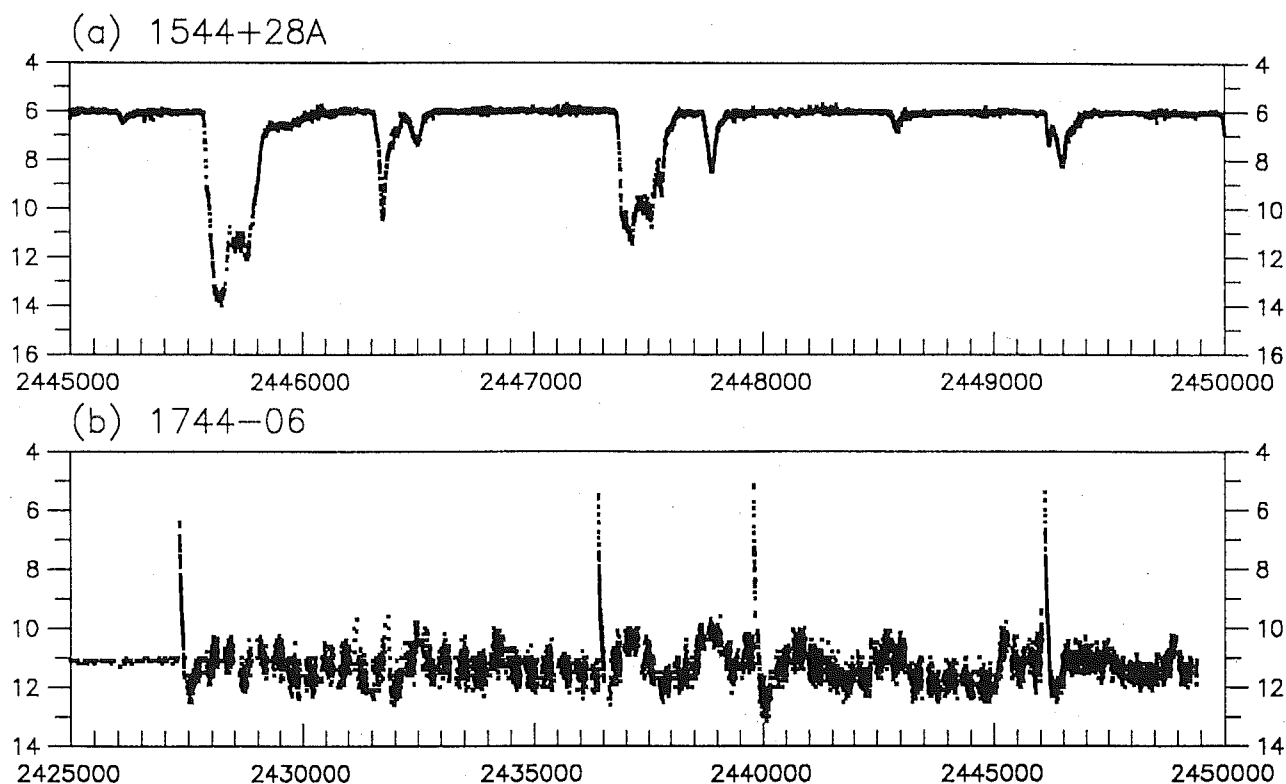
B. Rotating Variables

Rotating variables are rapidly rotating stars, often in binary systems, that undergo small-amplitude changes in light which may be due to dark or bright spots on the star's surface, similar to sunspots on our own Sun.

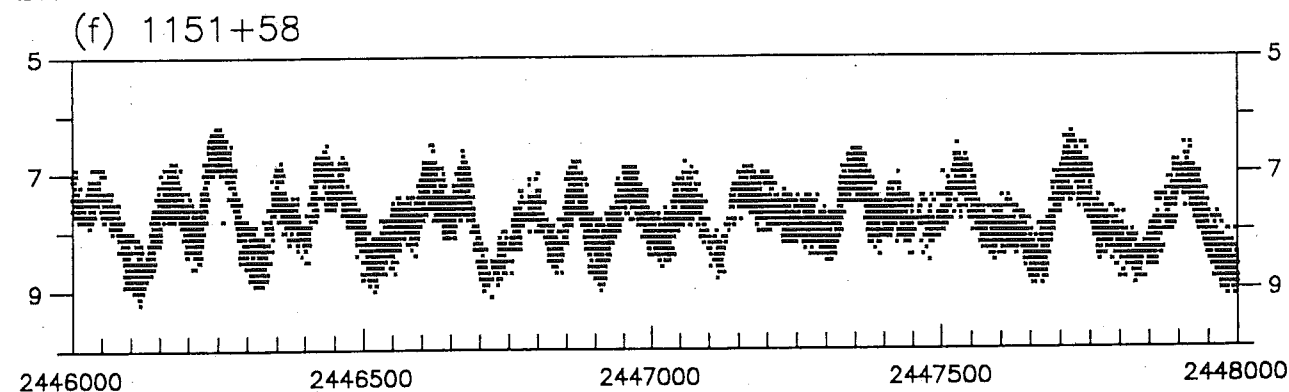
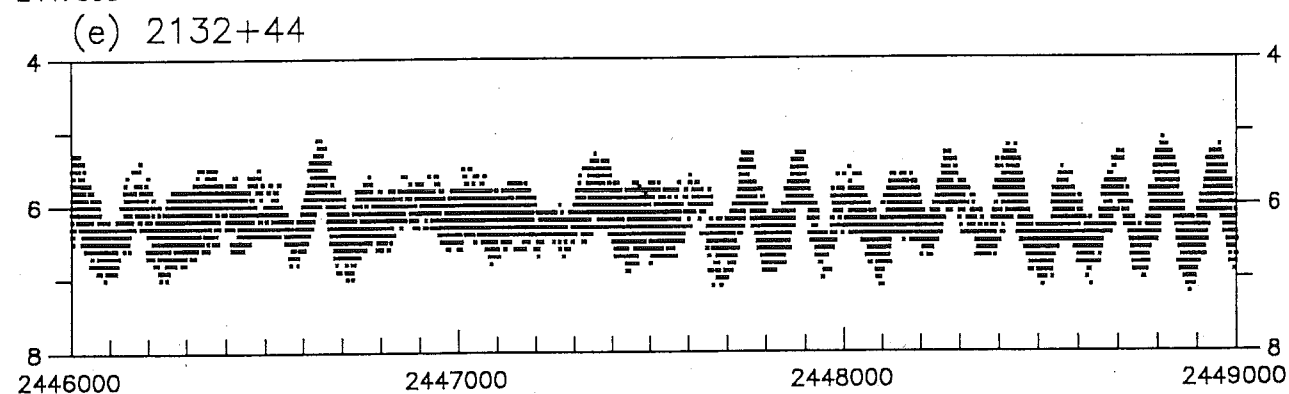
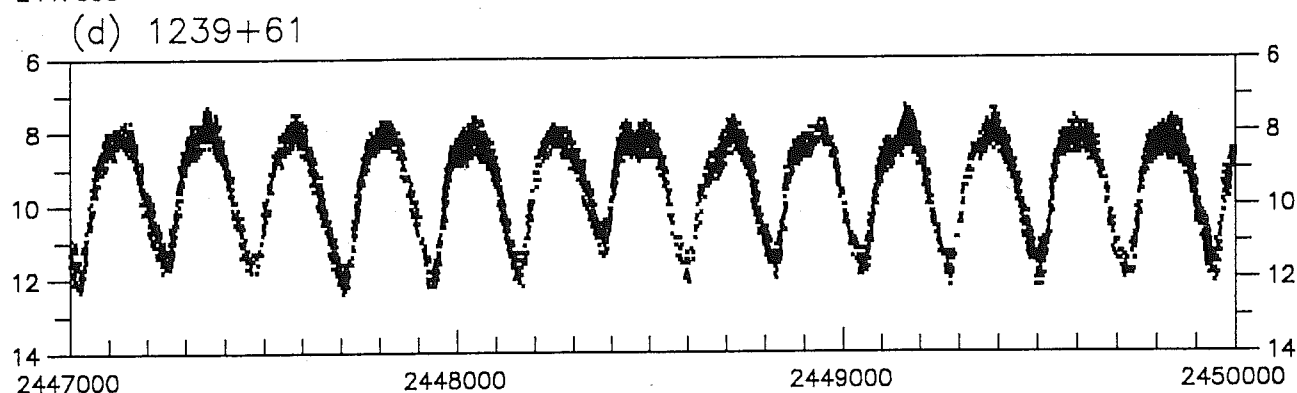
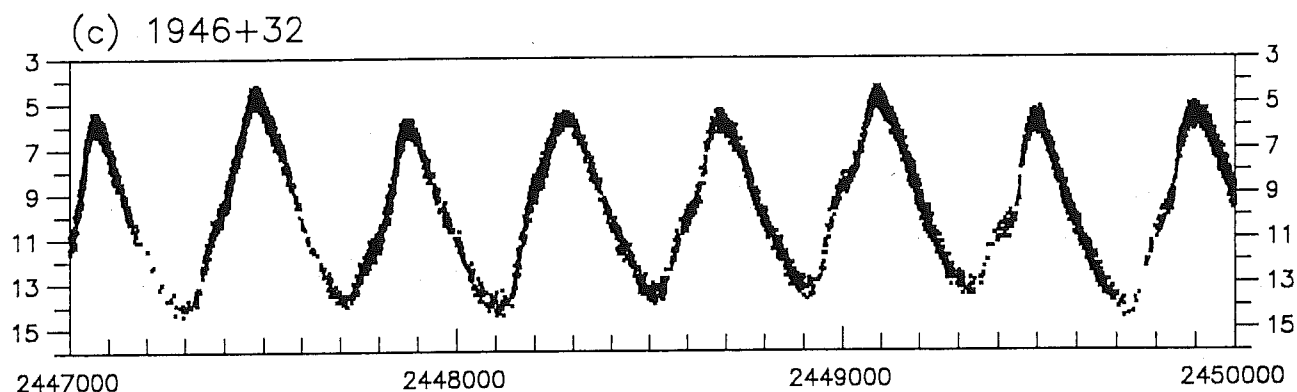
Chapter 11: Variable Stars, Light Curves, and Periodicity

Investigation 11.1: Recognizing Periodic Curves

A periodic curve is one which repeats identically within a fixed time interval. Study the following curves and determine which ones seem to exhibit periodic behavior. Determine the maxima, the minima, and the periods. From the description of the types of variable stars included with this investigation, what type(s) of variable star(s) do you think each light curve represents?

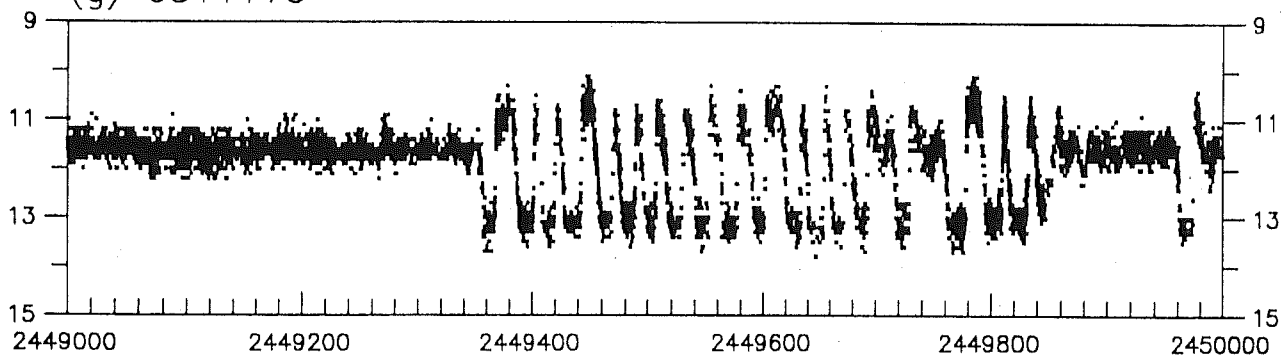


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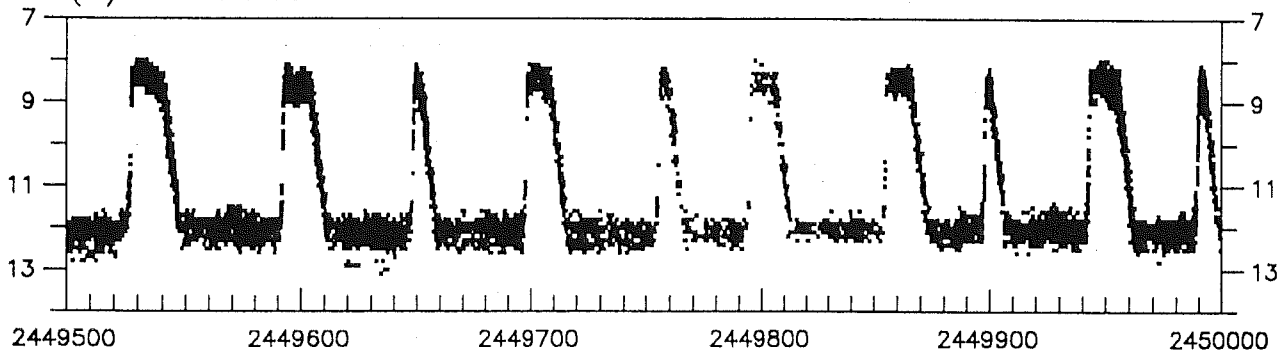


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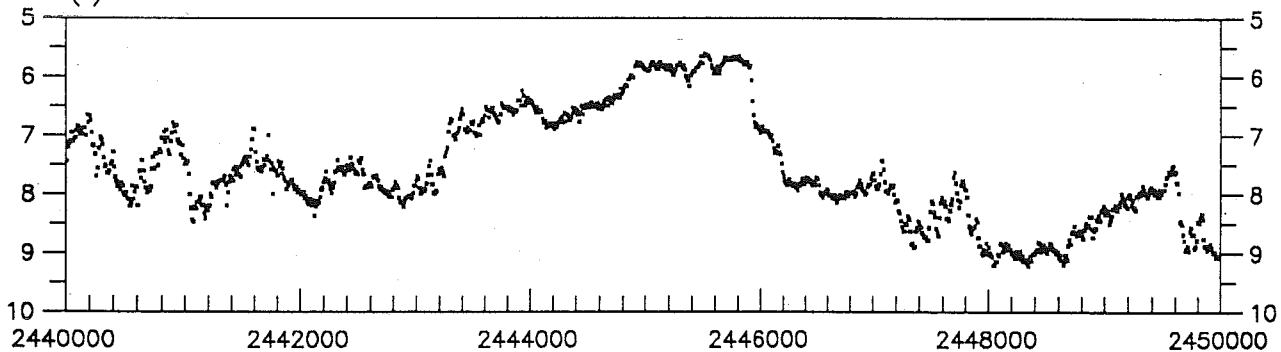
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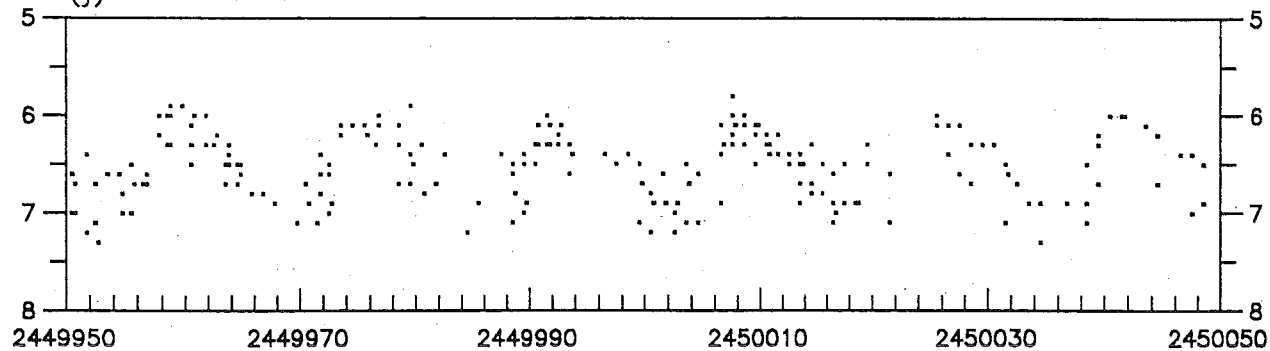
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(i) 1921+50

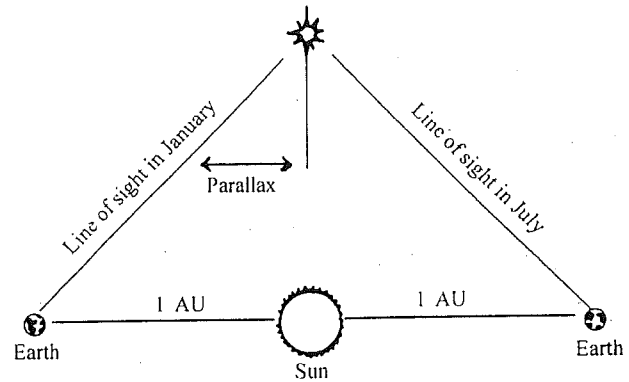


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Mapping the Universe (HIPPARCOS)

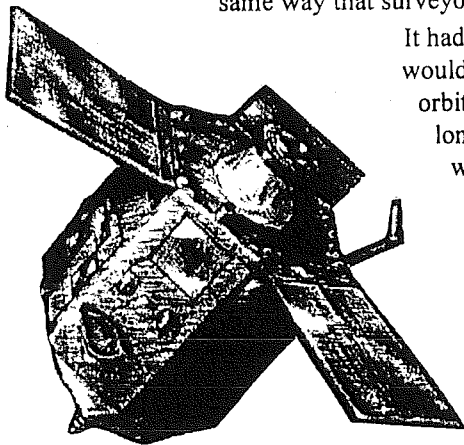
Astrometry is the science of measuring the positions of stars and other celestial objects utilizing a method called parallax. Parallax is the angular displacement in the apparent position of a celestial body relative to background stars when observed from two widely separated viewpoints. It is the angle that the baseline connecting the two viewpoints would subtend at the celestial object. From Earth, measurements are taken when the Earth is six months apart in its orbit around the Sun (see diagram) and the baseline is 2 AU. The angle is very small in value because the distance to other stars is so large, and is usually expressed in arcseconds (1 arcsecond = $1/3600$ th of one degree). The apparent shift for even the closest stars against the background stars is so minute that the maximum distance that can be measured directly by this method is very small. However, parallax measurements are the foundation of our knowledge of the cosmic distance scale and the age of the universe.



The High Precision PARallax Collecting Satellite (HIPPARCOS), launched on August 8, 1989, measured stellar positions, proper motions (the apparent angular motion per year of a star in right ascension and declination, due to its own actual movement through space and its motion relative to the Solar System), and parallaxes. From November 1989 to March 1993, HIPPARCOS systematically scanned the sky with instruments that used the same principles as ground-based parallax measurements to measure the apparent shifting of relatively nearby stars. The resulting more accurate data have been published by the European Space Agency in a 17-volume astrometric and photometric star catalogue, *The HIPPARCOS and TYCHO Catalogues*. The HIPPARCOS results will have a significant impact on the scientific community and set the standard for stellar distances and motions for decades to come.

More than 200 scientists selected 118,000 stars that were the targets for the mission. These included red and white dwarfs, giant stars, radio and X-ray stars, and variable stars, including binaries. Nearly 12,000 entries in the HIPPARCOS catalogue were of variable stars. Of those, 8,200 were not previously known to be variable. Among the 2,700 periodic variables listed, there were 273 Cepheids (2 new), 187 RR Lyrae stars (9 new), and 917 eclipsing binaries (343 new). HIPPARCOS measured these to a limiting magnitude of 12.5, an astrometric resolution of 0.001 arcsecond, and a photometric precision of 0.002 magnitude. These are the HIPPARCOS catalogue data. The mission also had a second objective, called TYCHO, to survey more than one million stars down to magnitude 11.5 with an astrometric resolution of 0.025 arcsecond and photometric precision of 0.06 magnitude. These are the TYCHO catalogue data.

HIPPARCOS scanned the entire sky continuously, systematically, and repeatedly with a telescope capable of accurately measuring the apparent shifts in angles between pairs of widely separated stars. The angles were measured by superimposing two fields of view approximately 58 degrees apart in the focal plane of a single telescope, each field containing one of the stars in a pair. This allowed the relative separations in one direction to be measured with very high accuracy. Other directions were then measured as the spacecraft rotated with respect to the sky. Repeated accurate recordings of star-pair separations made it possible to construct an accurate star position grid in much the same way that surveyors use triangulation techniques to pinpoint features on the Earth's surface.



Drawing of HIPPARCOS courtesy of European Space Agency

It had been planned that at the apogee of the initial orbit the apogee-boost motor would be fired, converting the elliptical transfer orbit into a circular geostationary orbit 36,000 km above the Equator, where the satellite would be maneuvered to a longitude of 12 degrees West, a location suitable for 24-hour per day contact with the ground control station at the European Space Operations Center (ESOC) in Germany. After the apogee-boost motor fired, the solar panels and communications antennae would be deployed and the satellite would start its slow scanning motion. However, the apogee-boost motor failed to fire, which left HIPPARCOS in a low elliptical orbit that took it through the Van Allen belts, exposing the solar panels to possibly serious radiation damage. Fortunately the panels were more resistant to radiation than was thought, and the satellite still managed to collect data 60 percent of the time in spite of the highly eccentric orbit, thereby allowing Hipparcos to exceed all expectations and produce the most accurate astrometric star catalogue to date.

Poster Page: From Hipparchus to HIPPARCOS

Hipparchus of Nicæa, a Greek astronomer who lived from 190 to 120 BC, first calculated the distance of the Moon from the Earth by measuring the Moon's parallax. Hipparchus also made the first star map and catalogue of 1,080 stars, which, when compared with observations made by his predecessors, led to the discovery of precession—the shifting of the Earth's rotational axis. This was achieved by measurements with the unaided eye, the resolution power of which is limited to a few minutes of arc. These early observations were rarely accurate to more than ~ 30 minutes of arc.

Little advance was made in astrometry until Tycho Brahe, using his brass quadrant and other recently invented instruments, carried out a long series of observations during the second half of the 16th century. His observations led to Kepler's Laws of planetary motion. In 1609, Galileo began using a new instrument for observations, the optical telescope, which led to a significant change for astrometry. The angular error in astrometric measurements fell to ~ 15 seconds of arc in the 1600's, and to ~ 8 seconds of arc by 1725.

Edmund Halley re-measured the rate of precession and compared the results with those of Hipparchus and others. While most stars displayed a general drift amounting to a precession of about 50 seconds of arc per year, Halley announced in 1718 that three stars, Aldebaran, Sirius, and Arcturus, were displaced from their expected positions by large fractions of a degree. Halley deduced that each star had its own "proper motion." Improvements in observational precision revealed the proper motions of many more stars, and in 1783 William Herschel found that he could partly explain these motions by assuming that the Sun itself was moving. This suggested that some stars might be relatively close to the Sun, and so astronomers intensified their efforts to detect "trigonometric parallax," the apparent oscillation in a star's position arising from the Earth's annual motion around the Sun.

Friedrich Bessel was the first to publish a parallax value for a star in 1838, following his studies of the motion of 61 Cygni. Bessel's careful analysis of the measurement errors and his use of both coordinates on the sky gave credibility to his results, after many previous claims from astronomers to have measured a stellar parallax. Thomas Henderson is credited with the first measurement of stellar parallax, that of the bright star Alpha Centauri, from observations made at the Cape of Good Hope, in 1832–33. Although he did not analyze the measurements for several years, the two components of this star, together with a faint companion called Proxima Centauri, form the nearest known group of stars to the Sun, at a distance of 4.2 light-years.

Observations improved substantially with the invention of photography. In 1887, a worldwide cooperative program among observatories from 18 countries, called "Carte du Ciel," used the same techniques and observing strategies to measure 13 million stars with a precision of ~ 1 arcsecond. Since then, determinations of photographic trigonometric parallaxes have been made at more than a dozen observatories. The technique is to measure the shift of the selected star relative to a few stars surrounding it on some 20 or more plates taken over a number of years. Several thousand parallaxes have now been measured from the ground, but only a few hundred parallaxes are considered to be known with an accuracy of better than ~ 20 percent of astrometric ground measurements. Any further progress on the ground was considered unlikely, since the most significant uncertainties remaining are caused by the Earth's atmosphere. In the early 1960's, some astronomers considered that the best prospect for major advances in measuring the positions and motions of stars was to go into space. The result was the launch of HIPPARCOS in 1989, with its precision in measurement in the range of milliarcseconds.

Chapter 11: Variable Stars, Light Curves, and Periodicity

Core Activity 11.2: Analyzing the Light Curve for Star X

You have already statistically analyzed the observational data for Star X. Now you will analyze the variation in magnitude for this star by plotting the data from Table 10.4, and drawing the light curve. You may have already drawn a rough light curve and estimated the period of a star, especially if you have observed delta Cep. We will now take a more systematic approach to the analysis of variable stars.

A. Plotting Individual Observational Data

1. A light curve is a plot of apparent magnitude versus time (JD). The magnitude is always plotted on the vertical (y) axis, with the brightest magnitude number (smallest number) at the top, and the dimmest magnitude number (largest number) at the bottom. Time, in the Julian Day unit, is always plotted along the horizontal (x) axis. Construct your graph with a scale that is proportional to your data, number and label the axes, and plot your observational data for Star X. (Use your own observational data, not the class average data.)
2. Draw a smooth “best-fit” curve through your data points. Remember not to connect the dots! The individual points are not important—the general trend of the points will give us the information we want about the star’s behavior.
3. Identify the regions on your graph where Star X is brightest and dimmest. The point at which a star is brightest is called its *maximum* (plural: maxima); its dimmest point is called the *minimum* (plural: minima).
 - a) What are the magnitude and the time of each maximum?
 - b) What are the magnitude and the time of each minimum?
4. A complete variation in light output of a variable star, from maximum to maximum or from minimum to minimum, is called a *cycle*. Has your star gone through one or more cycles? The length of the cycle (the difference between two successive minima or two successive maxima) is called the *period*. Using your light curve, find the period in days using two successive maxima or minima.

What is the period of Star X?

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B. Plotting Class Average Data

1. On the same graph, plot the class average values for Star X from Table 10.4 with a different symbol. Draw a smooth line through these points.
 - a) Do the class average values give the same value for magnitude and time of maximum and minimum? Why or why not?
 - b) Determine the period using two successive maxima or minima. Which result do you have more confidence in—your data or the class average data? Why?
2. In column [H] of Table 10.4, you recorded the standard error of the average for each data point. Using this information, draw error bars on all the points of the class average data in your graph. Remember, if the average for a given time is 3.5 and the standard deviation of the average is 0.032, then the true value should be 3.5 ± 0.032 magnitudes. (Since 0.032 goes to too many decimal places for your graph, simply drop the last digit.)

Are the error bars different sizes for each point?

3. To get an idea of the nature of the variability of the star, draw two smooth curves through the error bars, one through the tops of the bars, the other one through the bottoms of the bars. The class average values, then, will be between these two curves. The area inside these two curves is referred to as the *envelope*. The band, or envelope, represents 68% of the observations within the interval of average values you selected. The larger the envelope, the larger the spread in your data measurement, and therefore the less confidence there is in the average of the interval. Likewise, the smaller the envelope, the smaller the spread and the closer the data measurements are to each other. Therefore we can have more confidence in the average of the interval.
 - a) Compare your curve with the class average curve. Are there any significant differences?
 - b) Does your curve fall within the envelope?

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Activity 11.3: Analyzing the Light Curve for Delta Cephei

If you have classroom observational data for delta Cep and you have performed the necessary statistical analysis, plot the light curve and determine the periodicity for your data and the class average data. Construct the error bars and the envelope. Compare and discuss the results. Note: If you want to compare the results for Star X and delta Cep, or any other variable, remember that intercomparisons cannot be made unless the data are plotted on graphs with the same scale. This will make it easy for you to visually compare the differences and/or similarities of all stars you plot.

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Core Activity 11.4: Pogson's Method of Bisected Chords

So far you have determined the maxima and minima by visual inspection, which may have struck you as a not-so-very-scientific approach, especially since the difference between two successive maxima or minima is used to calculate the period, and this might not be adequate information for period determination. There is a more accurate method of making this determination, called *Pogson's method of bisected chords*. This method uses neither the high points (peaks) to determine maxima, nor the low points (dips) to determine minima. Instead, the total event, the variation of the star from beginning to end, is taken into consideration. An illustration of this method is shown in Figure 11.1 below for determination of the maximum magnitude.

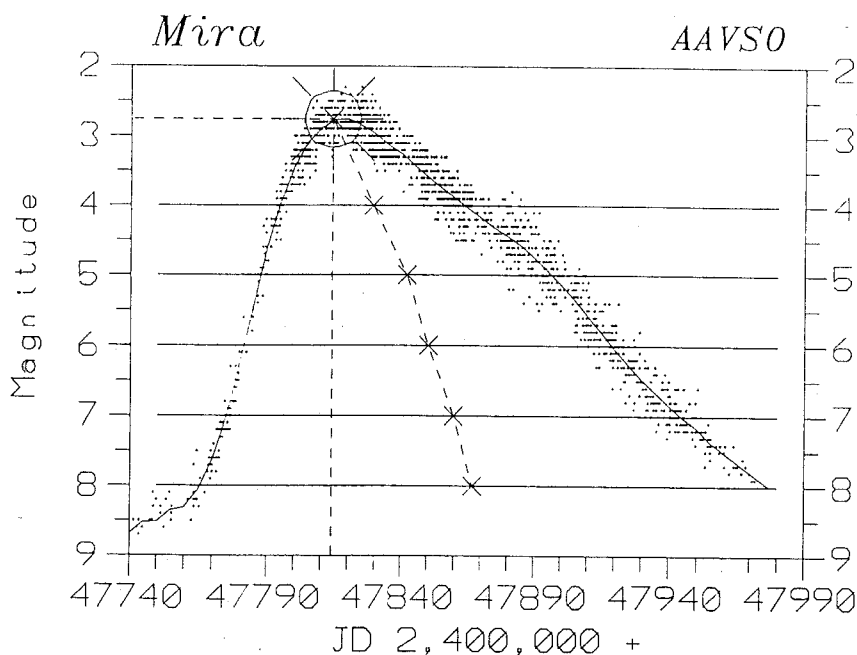


Figure 11.1

Chapter 11: Variable Stars, Light Curves, and Periodicity

To determine periodicity with Pogson's method:

1. Using the plotted graph with the best-fit smooth light curve of V Cas below (Figure 11.2), draw several straight lines (chords). The chords are drawn parallel to the x-axis and at various magnitudes through the rises and declines of the light curve. The midpoint of each chord is measured and marked (bisected). (Refer to Figure 11.1 on the preceding page.)
2. Draw a smooth line through the midpoints of the chords and continue it upwards until it reaches the light curve. Draw a perpendicular line from this point down to the x-axis. This is the date of maximum. Read across to the y-axis from the point at which the line intersected the light curve to find the magnitude at maximum.
3. To determine the date of minimum, the chords were drawn inside the peaks. Use the same method to determine the date of minimum, only now draw the chords inside the dips. Draw a smooth line through the midpoints of the chords, and continue it downwards until it reaches the light curve. Draw a perpendicular line from this point down to the x-axis to find the date of minimum. Read across to the y-axis from the point at which the line intersected the light curve to find the magnitude at minimum.
4. The line which bisects the midpoints may be a curve itself, and there may be observations which give a brighter or fainter magnitude, respectively, than the magnitude of maximum or minimum indicated by the intersection of the midpoint line with the light curve. That is, the magnitude at maximum may not be the brightest magnitude in the cycle, and magnitude at minimum may not be the faintest. After you have determined the maxima and minima dates, calculate the period.

NOTE: Since using Pogson's method to determine periodicity requires a large set of data, you will not be able to use Star X data for this activity, and probably not for class observational data of delta Cephei.

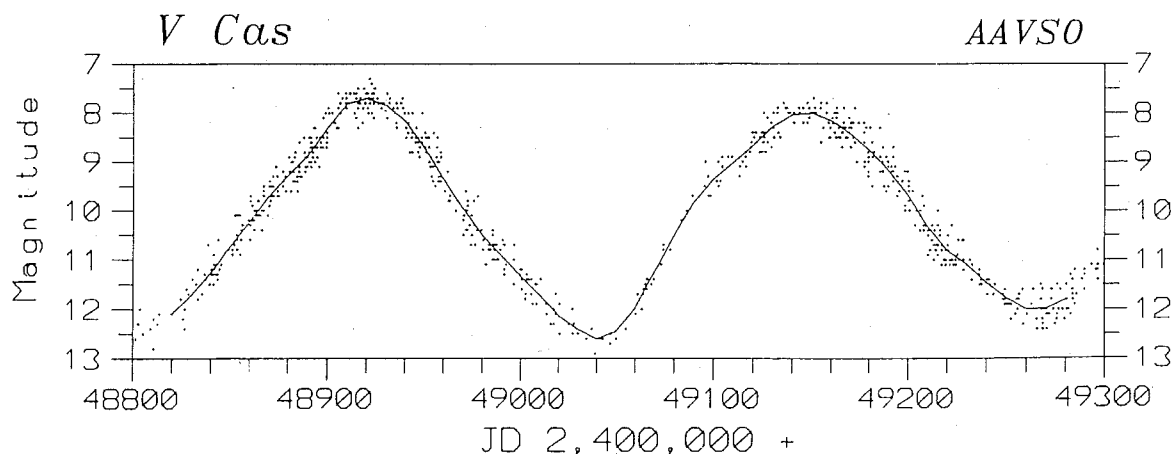


Figure 11.2

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Core Activity 11.5: VSTAR

VSTAR is a flexible software package developed at the AAVSO which enables you to plot light curves and analyze them in both simple and sophisticated ways. Your instructor will give you a copy of the user's manual. Read the instructions about how to start the program, how to load data, and how to change the view. Learn how to choose which time range of data to load, and learn how to change the limits of JD and magnitude in the on-screen plot.

VSTAR includes an archive of data from AAVSO observers for several stars. Let us use this program to study some of the data in its archive. Run the VSTAR program, and load data from the "ARCHIVE" file for the star V Cas. [NOTE: If more than one person or group of people runs the program at the same time over a computer network, make sure each is running in a different subdirectory. If two groups or individuals run the program in the same subdirectory, they will overwrite each other's log files.) Do not load all the available data; when it asks you for the "JD to start," enter 2447000, and when it asks you for the "JD to end," enter 2449000 (this will load 2,000 days of data, from JD 2447000 to 2449000). Look at the light curve; can you tell what kind of variable this is?

Estimating maxima by eye

You will notice that as you move the cursor around the screen, the program constantly displays the cursor location in time (JD) and magnitude. You can conveniently use this display to estimate the times and magnitudes of the maxima of V Cas.

Magnify the time scale so that you have a close-up look at the first maximum in these data. Place the cursor where you believe the maximum is located, both in time and in magnitude. If you hit the "P" key, VSTAR will record the time and magnitude of that point in its log file. VSTAR will also ask if you want to attach a comment to that record; you can just hit <ENTER> (no comment is necessary).

Now change the view so that you have a close-up look at the *next* maximum in these data. Again, place the cursor where you believe the maximum is located, and hit the "P" key, recording this information in your log file. Repeat this procedure for every maximum from JD 2447000 to 2449000. Then exit the VSTAR program.

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Now look at the file VSTAR.LOG (the log file created by VSTAR). It should have one line for each maximum, listing the time (JD) and magnitude of your estimate. For example, we found 9 maxima between JD 2447000 and 2449000:

2447125.5172	7.9500
2447351.7242	8.1500
2447571.7241	7.7000
2447795.8621	8.2500
2448020.0000	8.2500
2448228.2759	7.9000
2448473.7931	7.8000
2448692.4138	7.8000
2448921.3793	7.8000

VSTAR records both time and magnitude to the nearest 0.0001, but that is more accuracy than we can expect by eye, so we'll only use times to the nearest 0.01 day, and magnitudes to the nearest tenth.

We can estimate the period as the time from one maximum to the next, so for each maximum (except the first one), take its JD and subtract the JD of the previous maximum. With 9 maxima, this gives us 8 estimates of the period; our estimates are listed in the following table:

JD	Magnitude	Period
2447125.5172	7.9500	
2447351.7242	8.1500	226.21
2447571.7241	7.7000	220.00
2447795.8621	8.2500	224.14
2448020.0000	8.2500	224.14
2448228.2759	7.9000	208.28
2448473.7931	7.8000	245.52
2448692.4138	7.8000	218.62
2448921.3793	7.8000	228.97

Chapter 11: Variable Stars, Light Curves, and Periodicity

Now compute the average period, and the standard deviation. Finally, compute the standard deviation of the average. For our estimates, the average period is 224.48 days, with a standard deviation of 10.58, while the standard deviation of the average is 3.74.

Comparing one time span to another

Now repeat exactly the same procedure as above, with one exception: instead of studying the data from JD 2447000 to 2449000, study the data from JD 2440000 to 2442000. Estimate each maximum, and estimate the periods as the times from each maximum to the next. Compute the average period, the standard deviation, and the standard deviation of the average.

Is the period of V Cas the same from JD 2440000 to 2442000 as it was from JD 2447000 to 2449000?

Yet another way to estimate maxima

Now we will learn yet another way to estimate the time and magnitude of maximum: by fitting a polynomial. Once again, run the VSTAR program. Load the data for V Cas from JD 2447000 to 2449000. Change the view so that you have a close-up of the first maximum; adjust the “view window” so that your screen shows a single cycle of V Cas, starting at minimum, with maximum approximately in the middle of the screen, and ending with the next minimum.

Now hit the <F3> key (fit a polynomial). This instructs VSTAR to fit a smooth curve to the data, a curve of a type known as a *polynomial*. VSTAR will ask you for the polynomial degree, by prompting

Polynomial degree = 4

Change the “4” to “16” (so we can fit a 16th-degree polynomial), then hit <ENTER>. Now VSTAR will compute a 16th-degree polynomial fit to the data on your screen, and graph that smooth-fit curve as a red line. It will also show you a new menu at the bottom of the screen, the “polynomial menu.”

Now hit the <F5> key (locate MAX/min). VSTAR will ask you to place the cursor near the maximum/minimum you want to identify, then click the left mouse button (or keyboard equivalent). Place the cursor where you believe the maximum lies, then click the mouse button (or keyboard equivalent). VSTAR will compute the maximum or minimum which is closest to your cursor, and plot a blue cross at that location. It will also save the time and magnitude of that point to your log file. Now hit the <ESCAPE> key; this will take you out of the “polynomial menu” and back to the “main menu.”

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Repeat this procedure for every maximum from JD 2447000 to JD 2449000. Isolate each individual cycle (from minimum to minimum, with maximum approximately in the middle), fit a 16th-degree polynomial, and let VSTAR compute the time and magnitude of maximum. When you have finished, exit the VSTAR program.

Now look at the file VSTAR.LOG. It contains a lot of information you do not need, but it also has lines that look like this:

```
2447126.3306      7.8869 = MAX/min
```

This is the VSTAR estimate of the time and magnitude of maximum by polynomial fit. Remove from the file VSTAR.LOG all lines except these, leaving only the MAX/min estimates. We found 9 maxima between JD 2447000 and 2449000:

```
2447126.3306  7.8869 = MAX/min
2447352.5845  8.0180 = MAX/min
2447574.1225  7.6731 = MAX/min
2447798.9394  8.0078 = MAX/min
2448019.3119  8.0074 = MAX/min
2448231.7122  7.8539 = MAX/min
2448475.3867  7.8820 = MAX/min
2448694.9989  7.8019 = MAX/min
2448922.5602  7.6872 = MAX/min
```

Again, we can estimate the period as the time from one maximum to the next. So again, we have 8 estimates of the period of V Cas:

JD	Magnitude	Period
2447126.3306	7.8869	
2447352.5845	8.0180	226.2539
2447574.1225	7.6731	221.5380
2447798.9394	8.0078	224.8169
2448019.3119	8.0074	220.3725
2448231.7122	7.8539	212.4003
2448475.3867	7.8820	243.6745
2448694.9989	7.8019	219.6122
2448922.5602	7.6872	227.5613

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For each maximum (except the first), take the JD of maximum and subtract the JD of the previous maximum to get an estimate of the period. Compute the average period, the standard deviation, and the standard deviation of the average. For our estimates, the average period is 224.53 days, the standard deviation is 9.07, and the standard deviation of the average is 3.21 days.

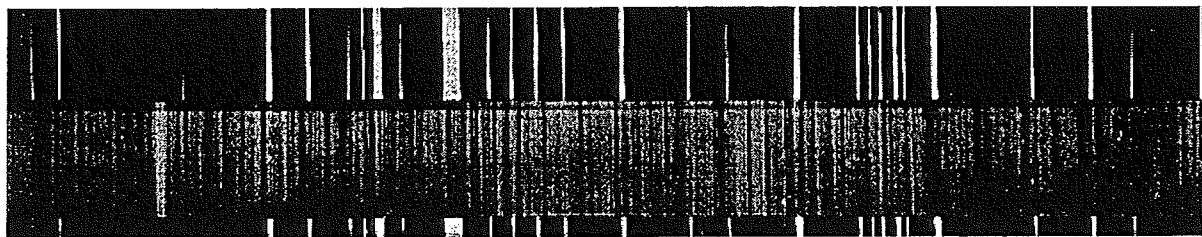
Now repeat exactly the same procedure as above, with one exception: instead of studying the data from JD 2447000 to 2449000, study the data from JD 2440000 to 2442000. Estimate each maximum by fitting a 16th-degree polynomial, and estimate the periods as the times from each maximum to the next. Compute the average period, the standard deviation, and the standard deviation of the average.

Is the period of V Cas the same from JD 2440000 to 2442000 as it was from JD 2447000 to 2449000?

Radar Guns and Speeding Stars

The frequency of a wave as it leaves its stationary source is the same as the frequency of the same wave being detected by an observer. If either the source or observer is moving, however, a perceived change in frequency occurs. We are familiar with the fact that horns and sirens of cars, trains, and ambulances have a higher pitch, or frequency, when they are approaching us, and a lower frequency when they are moving away from us.

The change in apparent frequency, and therefore wavelength, of a wave motion as a result of relative motion of the source and observer is known as the *Doppler effect*, and it works for any source which produces a wavelength, whether it is sound or electromagnetic radiation. As a source emitting a specific frequency moves forward, the wavelengths get “compressed” in front of the source and stretched out behind the source. For electromagnetic radiation emitted from a moving source, the amount of this change is known as the *Doppler shift*.



Doppler shift is apparent in the spectra of this star, compared to a reference spectra behind it.

For a source moving towards the observer, the observed wavelength is shorter than it would be if source and observer had no relative motion along the line joining them. This change to shorter wavelengths, towards the blue end of the visible spectrum, is called a *blueshift*. Conversely, if the source is moving away from the observer, there is a change to longer wavelengths, which is called a *redshift*.

When there is no relative motion between a source and the observer, the wavelength of a spectral line can be given by λ (the Greek letter *lambda*). If the relative velocity along the line of sight of source and observer is v , the change in wavelength $D\lambda$ of the spectral line is given by:

$$D\lambda / \lambda = v / c$$

where c is the speed of light. For values of v comparable with c , a relativistic expression is used:

$$D\lambda / \lambda = [(c + v) / (c - v)]^{1/2} - 1$$

where v is positive for a receding object and negative for an approaching object. Doppler shifts can be observed in all regions of the electromagnetic spectrum.

The Doppler effect has proven itself to be invaluable in the field of astronomy. The *radial velocity*, v_r , is the velocity of a star along the line of sight of an observer. It is calculated directly from the Doppler shift in the lines of the star's spectrum. If the star is receding, there will be a redshift in its spectral lines and the radial velocity will be positive. If the star is approaching, its motion will produce a blueshift and the velocity will be negative.

Similarly, the recessional velocities of *galaxies* can be determined from their redshifts. In this case, the emitting source—the galaxy—is moving away from the wavelengths of EMR it is emitting. The wavelengths become stretched out, producing a Doppler redshift in wavelength for observers on Earth.

The rate of this expansion for galaxies is related to the age of the universe. Nearby galaxies are moving away from us at speeds of about 250,000 m/s. Distant galaxies are moving away at speeds up to 90% of the speed of light.

The direction that galaxies are rotating can also be determined from Doppler shifts. The part turning towards Earth is blueshifted, and the part turning away is redshifted. The orbital motions of spectroscopic binaries and planetary systems are determined in a similar manner from shifts in spectral lines.

Poster Page: Radar Guns and Speeding Stars

Just as radar guns use the Doppler effect to determine the speeds of moving objects, spectroscopy likewise can tell us the speeds of distant stars and galaxies. Both operate on the same principle of perceived frequency and wavelength changes when emitting sources are moving relative to one another.

Another application of the Doppler effect, besides measuring the expansion of the universe and issuing speeding tickets, is in the medical profession. The Doppler effect forms the basis of a technique used to measure the speed of flow of blood. High-frequency sound waves called ultrasound are directed into an artery. The waves are reflected by red blood cells back to a receiver. The frequency detected at the receiver relative to that emitted by the source indicates the cell's speed. The reflected frequency is combined with some of the sound leaving the source and the combined waves produce a beat frequency which is easily measured, and gives the speed of the flow of the blood in the artery. This helps to determine the location of possible clots or advanced stages of hardening of the arteries. This is a similar arrangement to that used to measure the speed of cars, except that in radar, microwaves are used instead of ultrasound.

Meteorologists also use Doppler radar both in terrestrial and nonterrestrial applications. It is used to measure wind speeds involving severe weather, such as inside tornadoes. It has also helped determine the speeds of the winds on Mars and Venus. Instruments utilizing the Doppler effect are also used to help detect dangerous wind shear at airports.

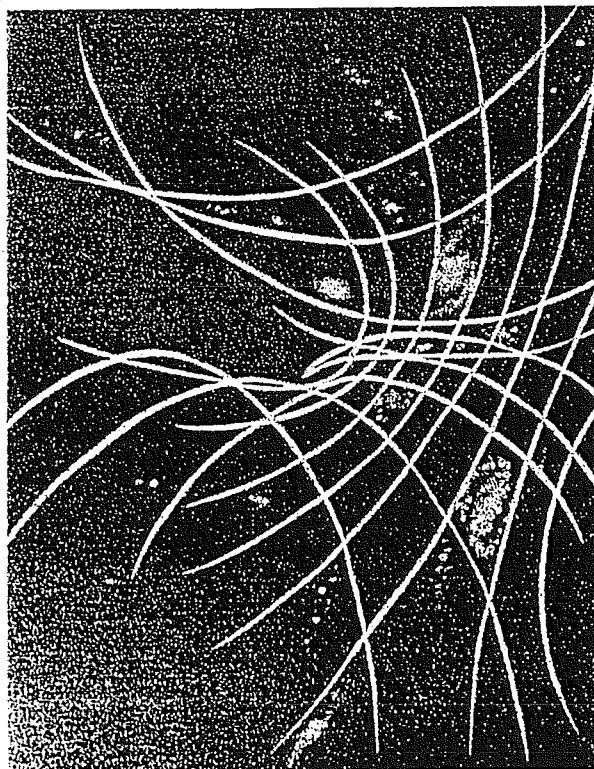
Sometimes it is surprising how a particular principle is applied or used over a wide range of applications. We are usually unaware of the ways that our lives are affected by principles that we often associate with one specific application. Many of us know that the Doppler effect is associated with radar guns and speeding tickets, but not that it helps measure weather, blood flow, and the age of the universe.

SPACE TALK

DI Herculis, a seemingly ordinary 8th-magnitude binary star 2,000 light-years from Earth, consists of two young blue stars separated by about one-fifth of the Earth-Sun distance. The two stars orbit their common center of mass, or **barycenter**, every 10.55 days. Studies of the DI Herculis eclipsing binary system have uncovered variations that are difficult to explain with either Newton's theory of gravitation or Einstein's theory of relativity. Because we see the system almost edge-on and the stars are small compared to their separation, DI Herculis exhibits very deep and sharp eclipses, and astronomers can make very precise timings. The eclipse times are used to analyze the orbital motions of the system. These motions are extremely puzzling.

According to Newton's theory of gravitation, isolated point particles circling one another will retrace the same paths forever. Real stars, however, are not points. They are big, have internal structure, and rotate. Also, their mutual gravities deform both stars. These departures from perfect symmetry cause the point at which the two stars are closest together, called **periastron**, to rotate about the system's barycenter. In other words, the stars do not retrace the same closed path: they follow an evolving rosette-like orbit, which continuously changes the times of the eclipses. However, the stars are far enough apart so that the gravity from one star does not deform the other into an egg-shaped object. This is important because any significant deviation from a spherical shape magnifies the tidal force and confuses the distinction between Newtonian effects and relativistic ones. DI Herculis' periastron is moving at a rate of $0.65'' \pm 0.18''$ every 100 years. This value is one-third of the expected rate of $1.93''$ per century, calculated using standard Newtonian physics.

There is an additional contribution to the periastron motion due to the curvature of space predicted by Einstein's **theory of general relativity**. The basic premise of general relativity is that gravitational fields change the geometry of spacetime, causing it to become curved. It is this curvature of spacetime that controls the natural motions of bodies. Matter tells spacetime how to curve and spacetime



Lola Chaisson, artist

"A Geometry For Spacetime"

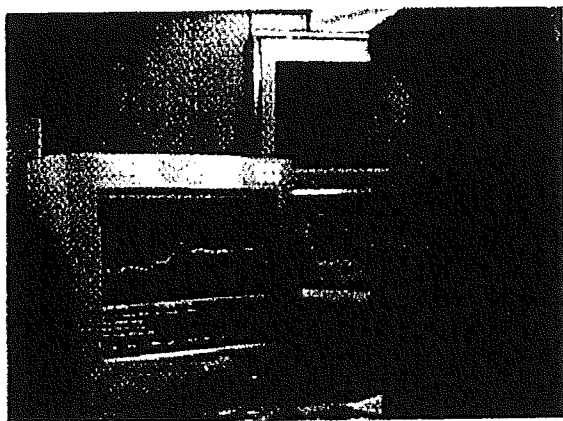
Chapter 11: Variable Stars, Light Curves, and Periodicity

tells matter how to move. General relativity may therefore be considered as a theory of gravitation; the differences between it and Newtonian gravitation only appear when the gravitational fields become very strong, as with black holes, neutron stars, and white dwarfs, or when very accurate measurements can be made with less-massive objects. When it was formulated by Einstein in 1915, general relativity explained a puzzling anomaly in the behavior of Mercury's orbit. Every planet traces an elliptical path around the Sun. The point where a planet comes closest to the Sun is called **perihelion**. Each planet's perihelion advances slightly from orbit to orbit because the other planets exert gravitational tugs. This **advance of perihelion** means a planet's orbit rotates with time, tracing a rosette pattern over the course of many revolutions. In the mid-1800's, it was obvious that Mercury's perihelion was advancing 43 arcseconds per century faster than Newton's laws predicted. Astronomers postulated that a planet closer to the Sun than Mercury was tugging on its orbit, a planet they named Vulcan. The search for Vulcan went on for years. Then Einstein's theory of general relativity predicted a curvature in the geometry of spacetime around the Sun that would significantly alter the trajectory of nearby Mercury, and the calculation was exactly 43". The mystery of Mercury was explained, and the search for Vulcan was discontinued.

The additional contribution to the periastron motion due to the curvature of space, predicted by the equations of general relativity for DI Herculis, amounts to a relativistic effect of $2^{\circ}34''$ for every 100 years. The net result is that the observed change is seven times smaller than was predicted using prevailing theoretical models. The discrepancy between observation and theory is well determined by several studies, and there is no problem with the observational data. Einstein's theory of gravity has passed previous observational tests with flying colors. However, there are a number of double stars for which it does not seem to work. DI Her should have provided an excellent test for general relativity in a strong gravitational field. DI Her's stars are massive enough (4.5 and 5.2 solar masses) and close enough that they significantly curve the spacetime around each other. The curvature should cause the system's **line of apsides**, the longest axis of the elliptical orbit, to advance 200 times faster than the Sun causes Mercury's perihelion to advance.

The possible causes for the discrepancy in **apsidal motion**—such as a third hidden star, tipped rotation axes, messy internal structures for the DI Her stars, unusual magnetic fields and extreme stellar winds—have all been ruled out. There is no evidence for orbital perturbation; the spectra show an upright orientation; no magnetic fields have been detected; and the observed winds are weak. DI Her is a mystery. No one is willing to abandon general relativity because it has more than proven itself. However, there may well be an addendum to the theory. One possibility is that spacetime has higher dimensions. General relativity works in four dimensions, but extending the theory to include five or more dimensions may explain why the perihelion advance of DI Her is much less than expected.

Chapter 12: Variable Stars and Phase Diagrams



Introduction

When the same cycle repeats over and over as regularly as clockwork, we refer to this as *periodic* behavior. If we want to know what is happening at any moment, it does not matter which cycle we are observing, because every cycle is exactly the same. What does matter is which *part* of the cycle we are observing. So if a star (or any other phenomenon) is perfectly

periodic, then its variation depends only on where it is in its cycle, a quantity called the *phase*.

A good example is an accurate clock. If it is a 24-hour clock (with an AM/PM indicator), it repeats exactly the same behavior, over and over, with a period of 1 day. Each day the clock goes through one cycle, and each cycle is just like every other cycle. If we want to know what the clock reads, we do not need to know which day it is (which cycle it is in), we just need to know the time of day (how far we are into the cycle).

Phase in Cycles

In the case of the clock, we might measure “how far into the cycle” it is in terms of hours and minutes, with the cycle starting at 00:00 and ending at 24:00. Of course, 24:00 (the end of the cycle) is also 00:00 on the *next day*, because the end of one cycle coincides with the beginning of a new one.

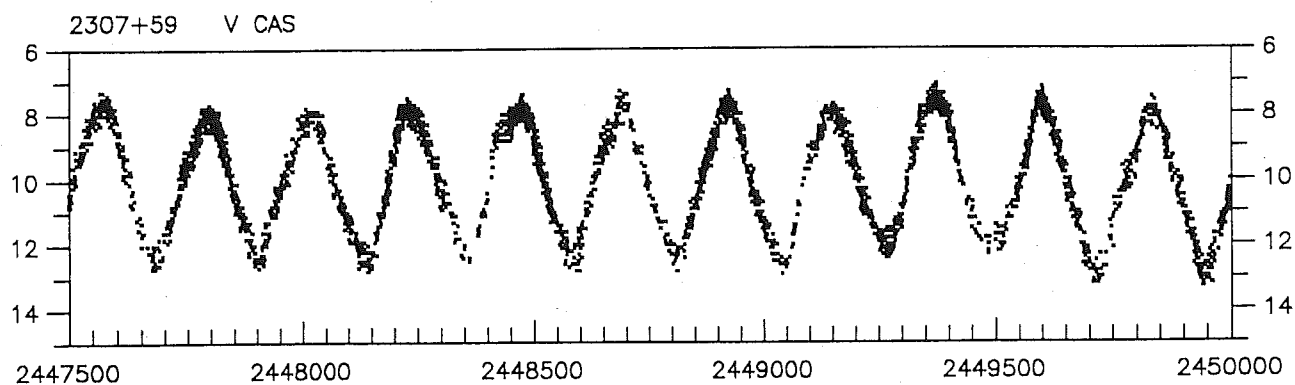
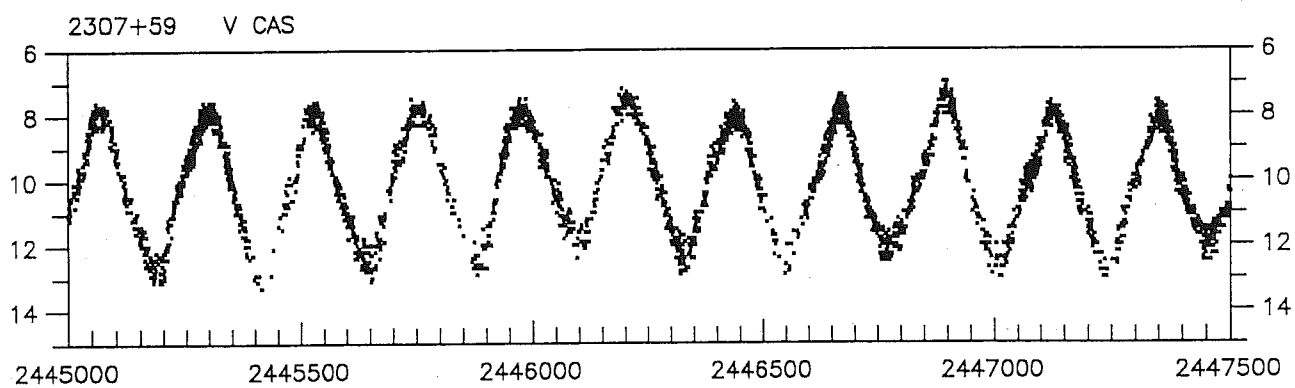
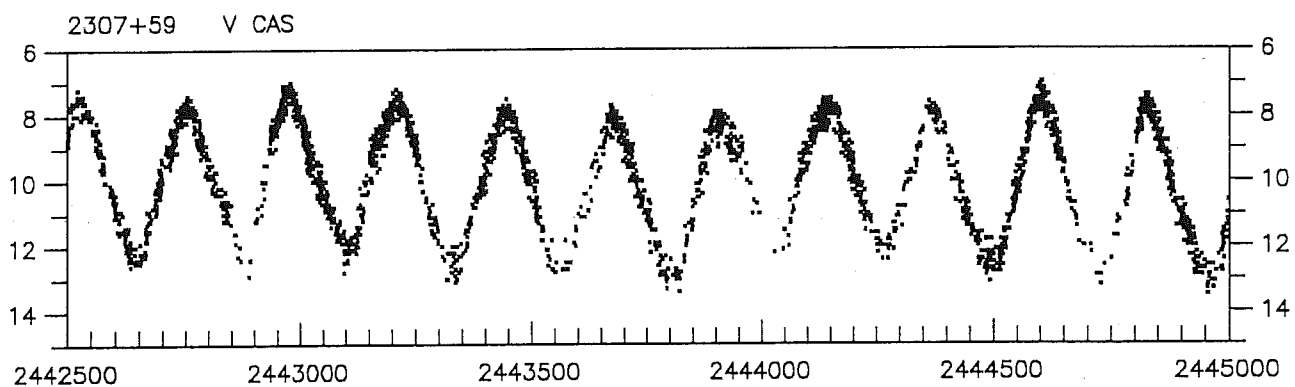
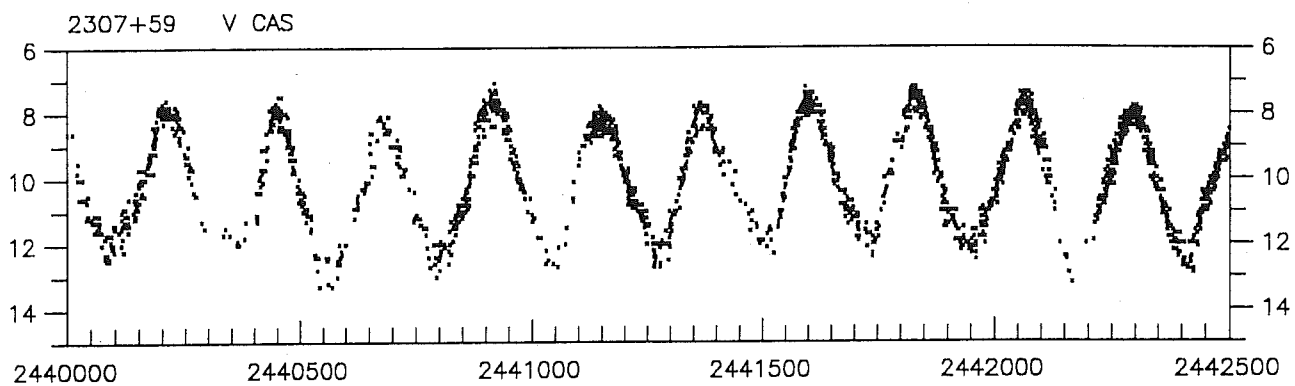
However, phase is measured in *cycles*, rather than in hours or minutes. Since phase is measured in cycles, of course a single cycle starts at 0 and ends at 1. In this case, the phase is simply the fraction of the cycle which has been completed so far. Thus a phase of 0.5 corresponds to 0.5 of the way (50%, or halfway) through the cycle, a phase of 0.2 is 20% (one-fifth) of the way through the cycle, etc. A phase of 1 is 100% of the way, the end of the cycle; it is also the beginning of the *next* cycle, so it is phase 0 of the next cycle.

To compute the phase in terms of cycles, we need to know how long each cycle is—in other words, we need to know what the *period* is. For the clock, we can express the phase in hours and minutes. But to express the phase in terms of cycles, we need to know that each cycle (each day) is 24 hours. That way, at noon, when we are 12 hours into the cycle, we know we are $12/24 = 0.5$ of the way through, meaning that the phase is 0.5.

Chapter 12: Variable Stars and Phase Diagrams

Investigation 12.1: Periodic Cycles

Cycles have no real beginning and no real end—they are continuous. Study the continuous light curve for V Cas on the opposite page. Cut out the sections and tape them together to see the behavior of V Cas from JD 2440000 to JD 2450000. If you use just two sections, do you still have a representation of the star's behavior? With just one section? Your instructor will give you a transparency of the same light curve. Can you cut the sections in any place and still have the same behavior pattern? Stack several sections over each other. Determine how small a segment is needed to give the same information and all four segments. What if you only had one cycle of the light curve? What if you start your cycle at maximum? Minimum? Describe your results.



A New Beginning

The clock cycle starts at midnight, because timekeepers have chosen to start each new day at midnight. But this is an arbitrary choice. Other cultures start a new day at sunrise or sunset, rather than at midnight. If the day (the cycle) starts at sunrise (say, 6:00 AM), then we are *not* halfway through the cycle at noon (12:00), so the phase is *not* 0.5. We may have 12 hours on the clock, but because we agree that our cycle *starts* at 6, we are only $12 - 6 = 6$ hours into the cycle. That is $6/24 = 0.25$ of the cycle (25%, or one fourth), so the phase is 0.25. Therefore, to compute phase we *also* need to know the starting time of the cycle. This is known as the *epoch*. For the clock, the epoch is usually midnight, but some people prefer to start their cycles at some other time.

These two quantities, the *period* and *epoch*, enable us to compute the *phase* at any given time. Suppose the epoch (start of the cycle) is at time t_0 , and the period is P . What is the phase at some other time t ? First we find how far we are into the cycle, by simply subtracting the starting time:

$$t - t_0.$$

This is the phase, in *time* units. To get the phase in units of cycles, we simply divide this by the period:

$$\phi = \frac{t - t_0}{P}.$$

The symbol ϕ is the Greek letter “phi,” which is used to represent the phase (in cycles). In the case of the clock, with the cycle starting at 6 AM, the period is $P = 24$ (hours) and the epoch is $t_0 = 6$. At noon ($t = 12$), the phase is:

$$\phi = \frac{t - t_0}{P} = \frac{12 - 6}{24} = \frac{6}{24} = 0.25.$$

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0 = 1 (yes, zero equals one)

Let's go back to starting each new day at midnight, so for our clock the period is $P = 24$ and the epoch is $t_0 = 0$. At midnight ($t = 0$), the phase is 0 (start of the cycle), at noon ($t = 12$) the phase is 0.5 (halfway), and at the following midnight ($t = 24$) the phase is 1 (end of the cycle). What about the *following* noon?

In this case the time is $t = 36$; it has been 36 hours since our "epoch." The phase is:

$$\phi = \frac{36 - 0}{24} = 1.5.$$

But something is not right here. We said that phase was "how far along we are in the cycle," and that it did not matter which cycle, so it should be the same, every noon. But this phase ($\phi = 1.5$) is *not* the same as that of the previous noon ($\phi = 0.5$).

Or is it? We could say that $\phi = 1.5$ is "one-and-a-half cycles," or we could say that it is "halfway through the *next* cycle." Since we are not interested in which cycle, we ignore the "next" part, and say "halfway"; thus the phase is 0.5.

In fact, whenever we compute a phase, we can make it into a "standard" phase by simply *ignoring* which cycle. If the phase is $\phi = 3.11$ (a little more than three cycles), we are 11% of the way through *three cycles later*. We will ignore the "three cycles later" part, note that we are 11% of the way through a cycle, and say the phase is 0.11. All *standard* phases are between 0 and 1.

When phase is expressed in cycles, it is easy to identify which cycle; it is just the *integer* part of the phase, or cycle number. For a phase $\phi = 3.11$, the integer part (3) tells us that we are dealing with cycle 3, and 0.11 of 3.11 tells us we are partway through cycle 3. Since a standard phase ignores this, we can simply ignore the integer part of the phase (telling us which cycle). What really counts is the *decimal* part of the phase (how far into the cycle). So we will modify the above equation, and say that the (standardized) *phase* is the decimal part of what we had before:

$$\phi = \text{decimal part of } \left[\frac{t - t_0}{P} \right].$$

What this means is that a phase of 1 (start of next cycle) is really the same as a phase of 0 (start of this cycle) or a phase of 3, or 17, or 256. Any two phases which differ by an integer are really the same phase. Phase 1.5 is the same as phase 0.5, phase 12.336 is the same as phase 0.336, and yes, $0 = 1$.

This may seem strange, but it is actually mathematically sound. We are simply taking all numbers *modulo 1*. We can still do arithmetic, still compute numbers, but we always ignore the integer part of whatever we end up with. This kind of arithmetic is known as

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modular arithmetic. In modular arithmetic, modulo 1, we can quite validly state the following equation:

$$0 = 1 = 2 = 3 = 4 = \dots$$

Negative Phases

One thing to be careful of is *negative* phases. For the clock example ($P=24$ and $t_0=0$), let us compute the phase at 6 AM three days *previously*. In this case the time is $t = -66$ (it is 66 hours before our epoch), so the phase is:

$$\phi = \frac{-66-0}{24} = -2.75$$

To convert this to a standard phase, we cannot “just ignore” the -2 and call it 0.75. If we did that, we would also be ignoring the minus sign. So we will just remember that we are doing arithmetic modulo 1, which allows us to add or subtract *any* integer without really changing the result. Let us add 3: $\phi = -2.75 + 3.00 = 0.25$. This does fall in the range 0 to 1, so this is the standard phase.

Folded Light Curves

Take a look at the following light curve of the Cepheid-type variable X Cyg (Figure 12.1). All observations were made by AAVSO observer LX. There are enough data that we can see an up-and-down variation, which turns out to be periodic with a period of 16.285 days. Still, there are only a few observations for each cycle, so it is difficult to tell exactly what the *shape* of a cycle is.

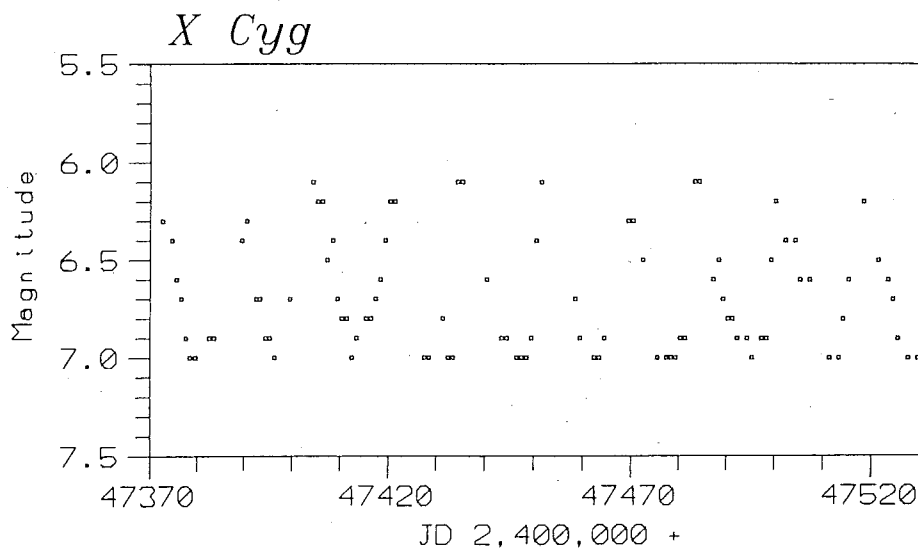


Figure 12.1

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It would be nice if we could superimpose all the cycles on top of each other. We would like to plot each data point, but instead of plotting the time, we would like to plot “how far it is into the cycle.” That way, all the cycles will be “folded” on top of each other, and we may have enough data to give us an accurate picture of what the cycle looks like. We already have a name for “how far into the cycle”: we call it the *phase*. For a variable star, we can do exactly the same thing we did with the clock. Find the period P , choose an epoch t_0 , and we can compute the standard phase for any time t . Then we can plot a light curve, but instead of plotting magnitude as a function of *time*, we will plot magnitude as a function of *phase*. This will give us what is called a *folded light curve*, or *phase diagram*.

Let’s use the period 16.285 days, and choose as a starting point JD 2,447,400 (an arbitrary choice). Then we can take each observation and convert the time into phase. Plotting brightness as a function of phase, we have the following folded light curve (Figure 12.2):

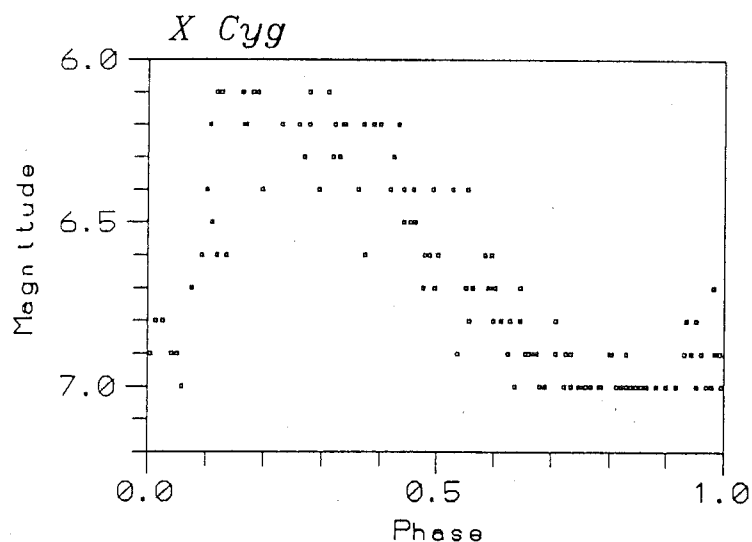


Figure 12.2

Now we *can* see what the shape of the cycle is. There is a very rapid rise from minimum to maximum, followed by a much slower decline from maximum to minimum.

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Core Activity 12.2: Folded Light Curve of the Variable Star SV Vul

You are now ready to construct a folded light curve, or phase diagram, using the observations of a Cepheid variable given in the following table.

Table 12.1
SV Vul Magnitude Measurements (1987 – 1989)

Julian Date	Magnitude	Julian Date	Magnitude
2447011.6	7.0	2447458.5	6.9
2447023.6	7.5	2447475.5	7.8
2447040.6	7.9	2447492.5	7.9
2447066.5	7.4	2447505.5	7.2
2447091.4	7.0	2447529.5	7.9
2447103.6	7.2	2447707.6	7.9
2447124.6	7.8	2447722.6	6.7
2447171.5	7.9	2447747.6	7.8
2447308.6	7.9	2447769.5	6.8
2447338.6	7.8	2447778.5	7.1
2447374.6	7.0	2447800.5	7.9
2447390.6	7.9	2447821.6	7.0
2447404.5	7.8	2447832.5	7.5
2447413.5	6.8	2447848.5	7.9
2447421.5	7.2	2447857.4	6.8
2447444.5	7.9	2447868.5	7.2

1. Construct a graph with the magnitude on the vertical axis and phase on the horizontal axis. Determine the appropriate magnitude scale from the data in Table 12.1. Since all standard phases are between 0 and 1, choose a scale for the phase axis which goes from 0 to 1.
2. We defined the phase as the decimal part of $[(t-t_0)/P]$, where t_0 is the epoch and P is the period. Take the JD of the very first observation as the epoch, so $t_0 = 2447011.6$. Then the first observation occurs at the start of the cycle (we chose our epoch that way), so we already know the phase of the first

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observation: it is at phase 0 (start of the cycle). The magnitude of the first observation is 7.0, so plot a point on your graph at phase 0 and magnitude 7.0.

3. For all the other observations, we apply our formula for computing phase. First we take the time of the observation and subtract the epoch time t_0 . For the 2nd data point, this gives

$$\begin{array}{r} 2447023.6 \text{ (time of observation } t) \\ - 2447011.6 \text{ (time of epoch } t_0) \\ \hline 12.0 \text{ (time difference)} \end{array}$$

Then we divide by the period P . For SV Vul, the period is $P = 44.8$ days. This gives

$$12.0 / 44.8 = 0.2679$$

Then we take the decimal part of what we get. Since this result is already between 0 and 1, it is already a standard phase. So for observation #2, the phase turns out to be 0.2679. For plotting purposes, we can round this off to 0.27.

Repeat this process for every data point, computing the standard phase. When you have computed all the phases, plot each data point at the correct phase and magnitude.

4. Draw a smooth curve showing the trend of the data. Do most of the data lie near this smooth curve? This is a test of the period. Lots of scatter with no obvious trend would show that the measured period is not correct. The correct period should produce a phase diagram whose scatter is about the same as the scatter in the raw data (usually about 0.2 magnitude).

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Double Your Fun

Look again at the folded light curve of X Cyg (Figure 12.2). It is a little difficult to see the behavior near minimum, because the picture is “broken” at phase 0 = 1, leaving a gap in the graph. It would be nice if we had a clear picture of the entire cycle, with no breaks.

We can, if we use the fact that phase is a modular quantity, modulo 1. So a phase of 0 is the same as a phase of 1, and the same as a phase of -1. A phase of 0.133 is the same as a phase of $0.133 - 1 = -0.867$. A phase of 0.58 is the same as $0.58 - 1 = -0.42$. For each time, let us compute not just one phase, but *two* phases. We will compute the standard phase, which is always between 0 and 1, and we will also compute the “previous cycle phase (ϕ')”, which will be between -1 and 0. If the standard phase is ϕ , then the “previous cycle phase” is $\phi' = \phi - 1$.

Note that this “previous-cycle phase” will end up being negative. We already learned how to change a negative phase into a standard phase. Now we are changing a standard phase into a negative phase! But not just any negative phase will do. A proper “previous-cycle phase” has to fall in the range of -1 to 0, just as a proper “standard phase” must fall in the range 0 to 1. It is easy to compute, if you just remember to take the standard phase and subtract 1 to give the proper “previous-cycle” phase. **Always compute the standard phase first, then subtract 1 to get the previous-cycle phase.**

We will plot magnitude as a function of phase, but we will plot each data point at *both* phases, the standard phase and the “previous-cycle” phase. In effect, we will be plotting each data point *twice*, and since our phases now run from -1 to +1, we will have a nice picture of not one, but *two* complete cycles. Now it is easy to see what the star is doing at any point of its cycle, because we have an unbroken graph (Figure 12.3):

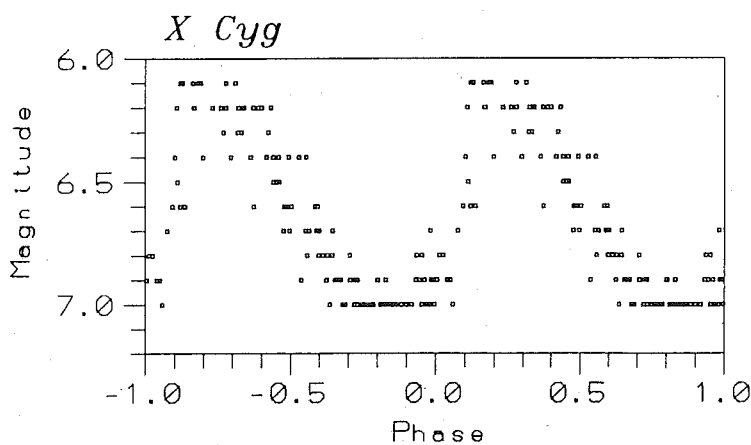


Figure 12.3

When astronomers plot folded light curves, they almost always plot two complete cycles, with phase extending from -1 to +1, in order to give a clear picture of the shape of the entire cycle.

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Core Activity 12.3: Another Folded Light Curve of SV Vul

Table 12.1
SV Vul Magnitude Measurements (1987 – 1989)

Julian Date	Magnitude	Julian Date	Magnitude
2447011.6	7.0	2447458.5	6.9
2447023.6	7.5	2447475.5	7.8
2447040.6	7.9	2447492.5	7.9
2447066.5	7.4	2447505.5	7.2
2447091.4	7.0	2447529.5	7.9
2447103.6	7.2	2447707.6	7.9
2447124.6	7.8	2447722.6	6.7
2447171.5	7.9	2447747.6	7.8
2447308.6	7.9	2447769.5	6.8
2447338.6	7.8	2447778.5	7.1
2447374.6	7.0	2447800.5	7.9
2447390.6	7.9	2447821.6	7.0
2447404.5	7.8	2447832.5	7.5
2447413.5	6.8	2447848.5	7.9
2447421.5	7.2	2447857.4	6.8
2447444.5	7.9	2447868.5	7.2

You are now ready to construct a second phase diagram of SV Vul, this time computing two phases for each point, and plotting a folded light curve showing two complete cycles.

1. Construct a graph with magnitude on the vertical axis and phase on the horizontal axis. Determine the appropriate magnitude scale from the data in Table 12.1. Since we will be plotting two cycles in our folded light curve, our phases will run from -1 to +1. Choose a scale for the phase axis which goes from -1 to 1.
2. We defined the phase as the decimal part of $[(t-t_0)/P]$, where t_0 is the epoch and P is the period. Take the JD of the very first observation as the epoch, so $t_0 = 2447011.6$. Then the first observation occurs at the start of the cycle (we

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chose our epoch that way), so we already know the phase of the first observation: it is at phase 0 (start of the cycle). The magnitude of the first observation is 7.0, so plot a point on your graph at phase 0 and magnitude 7.0.

3. For all the other observations, we apply our formula for computing phase. First we take the time of the observation and subtract the epoch time t_0 . For the 2nd data point, this gives

$$\begin{array}{rcl} 2447023.6 & \text{(time of observation } t) & \\ - 2447011.6 & \text{(time of epoch } t_0) & \\ \hline 12.0 & \text{(time difference)} & \end{array}$$

Then we divide by the period P . For SV Vul, the period is $P = 44.8$ days. This gives

$$12.0 / 44.8 = 0.2679$$

Finally, we take the decimal part of what we get. Since this result is already between 0 and 1, it is already a standard phase. So for observation #2, the phase turns out to be 0.2679. For plotting purposes, we can round this off to 0.27.

Repeat this process for every data point, computing the standard phase for each data point.

4. Now compute the “previous cycle phase” for each data point. To do so, simply take the “standard phase” you just computed, and subtract 1. You now have two phases for each data point, a standard phase between 0 and 1, and a previous-cycle phase between -1 and 0.
5. Plot each data point at its correct magnitude, and at *both* phases (so each observation gives two points on the graph).
6. Draw a smooth curve showing the trend of the data. You should be able to discern two complete cycles of variation in the graphs. Do most of the data lie near this smooth curve? This is a test of the period. Lots of scatter with no obvious trend would show that the measured period is not correct. The correct period should produce a phase diagram whose scatter is about the same as the scatter in the raw data (usually about 0.2 magnitude).

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Start from the Top

For X Cyg in Figure 12.1, we chose as our epoch, or starting point, JD 2,447,400, just because it was convenient. However, astronomers prefer to choose an epoch so that the *maximum occurs at phase zero*. So let us take as our epoch the time of one of the maxima, JD 2,447,403.0. Then our folded light curve has its maximum right at phase 0 (Figure 12.4):

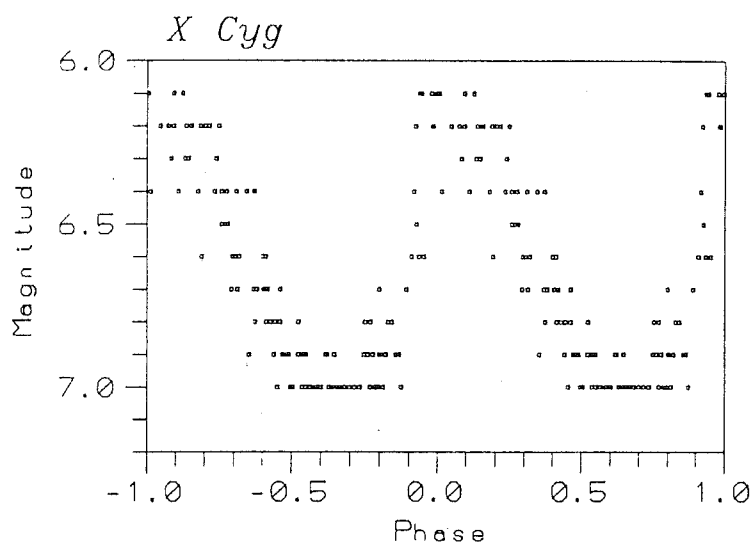


Figure 12.4

This is the *standard folded light curve* for a variable star. It plots two complete cycles, with phase running from -1 to +1, and the epoch is chosen so that maximum occurs at phase zero.

There is one exception to this rule: eclipsing binary stars. For eclipsing binaries, the *minimum* brightness (which usually occurs in the middle of the eclipse) is the part we are really interested in, so we choose the epoch so that phase zero is *minimum* rather than maximum.

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Core Activity 12.4: Yet Another Folded Light Curve of SV Vul

Table 12.1
SV Vul Magnitude Measurements (1987 – 1989)

Julian Date	Magnitude	Julian Date	Magnitude
2447011.6	7.0	2447458.5	6.9
2447023.6	7.5	2447475.5	7.8
2447040.6	7.9	2447492.5	7.9
2447066.5	7.4	2447505.5	7.2
2447091.4	7.0	2447529.5	7.9
2447103.6	7.2	2447707.6	7.9
2447124.6	7.8	2447722.6	6.7
2447171.5	7.9	2447747.6	7.8
2447308.6	7.9	2447769.5	6.8
2447338.6	7.8	2447778.5	7.1
2447374.6	7.0	2447800.5	7.9
2447390.6	7.9	2447821.6	7.0
2447404.5	7.8	2447832.5	7.5
2447413.5	6.8	2447848.5	7.9
2447421.5	7.2	2447857.4	6.8
2447444.5	7.9	2447868.5	7.2

You are now ready to construct a “standard” phase diagram. Estimate the maximum by inspecting the data in Table 12.1, and choose an epoch so that the maximum occurs at phase 0.

1. Construct a graph with magnitude on the vertical axis and phase on the horizontal axis. Determine the appropriate magnitude scale from the data in Table 12.1. Since we will be plotting two cycles in our folded light curve, our phases will run from -1 to +1. Choose a scale for the phase axis which goes from -1 to 1.
2. We defined the phase as the decimal part of $[(t-t_0)/P]$, where t_0 is the epoch

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and P is the period. The brightest of all the observations is the estimated magnitude of 6.7 on JD 2447722.6. Take this as a rough estimate of the time of maximum, and use it as your epoch: $t_0 = 2447722.6$.

3. For each observation, apply the formula for computing phase. First we take the time of the observation and subtract the epoch time t_0 . For the 1st data point, this gives

$$\begin{array}{rcl} 2447011.6 & \text{(time of observation } t) & \\ - 2447722.6 & \text{(time of epoch } t_0) & \\ \hline -711.0 & \text{(time difference)} & \end{array}$$

Then we divide by the period P . For SV Vul, the period is $P = 44.8$ days. This gives

$$-711 / 44.8 = -15.8705$$

Finally, we take the decimal part of what we obtain. Since this result is negative, we remember to add an integer to make the sum fall between 0 and 1. Adding 16, we get the standard phase as $\phi = 0.1295$. Repeat this process for every data point, computing the standard phase.

4. Now compute the “previous cycle phase” for each data point. To do so, simply take the “standard phase” you just computed, and subtract 1. You now have two phases for each data point, a standard phase between 0 and 1, and a previous-cycle phase between -1 and 0.
5. Plot each data point at its correct magnitude, and at *both* phases (so each observation gives two points on the graph). This is the standard folded light curve.
6. Draw a smooth curve showing the trend of the data. You should be able to discern two complete cycles of variation in the graphs. Does the maximum lie at phase 0? This is a test of the epoch; if the maximum is noticeably different from phase 0, then the epoch is not quite correct. Do most of the data lie near this smooth curve? This is a test of the period. Lots of scatter with no obvious trend would show that the measured period is not correct. The correct period should produce a phase diagram whose scatter is about the same as the scatter in the raw data (usually about 0.2 magnitude).

The Discovery of SS Cygni

(Adapted from a paper entitled "The Centennial of the Discovery of SS Cygni" by Martha L. Hazen, published in the *Journal of the AAVSO*, Volume 26, 1997, pp. 59–61. Dr. Hazen is the Curator of Astronomical Photographs at Harvard College Observatory. Additional information about SS Cygni was provided by the technical staff of the AAVSO.)

In the *Harvard College Observatory Circular* No. 12, signed by Edward C. Pickering and dated November 2, 1896, there appeared a listing entitled "New Variable Stars in Crux and Cygnus." The last paragraph of the listing reads:

In addition to the above objects a star in the constellation Cygnus, whose approximate position for 1900 is R.A. = $21^{\text{h}}38^{\text{m}}.8$, Dec. $+43^{\circ}8$ has been found to be variable by Miss Louisa D. Wells. Its period appears to be about 40 days and its photographic brightness varies from 7.2 to fainter than 11.2, an unusually large range for a variable having so short a period.

Miss Louisa D. Wells was one of the women "computers" hired by E.C. Pickering to work with the Harvard photographic plates. All the original research notebooks kept by the "computers" are stored at the Harvard University Deposit Library, and a complete list of the notebooks is available at the Plate Stacks of the Harvard Observatory. Curiously, Miss Wells's notebooks, necessary for her work, are not in the collection. Her supervisor at the time made no mention of the discovery.

The earliest plate intentionally taken of SS Cyg is plate I 15990, taken with the 8-inch Draper refractor located on the grounds of the Observatory in Cambridge, MA. The plate was taken on September 23, 1896, according to the record book. The entry in the "Object" column says "Susp. var." and the first word is crossed out and "L.D.W.'s" written above.

The earliest plate still extant was taken on September 24, 1890 (see Figure 1a; a hand-drawn arrow points to SS Cyg in roughly the center of the photo), when SS Cyg was at or near minimum. Figure 1b is a plate taken on October 30, 1890, when the star was in outburst.

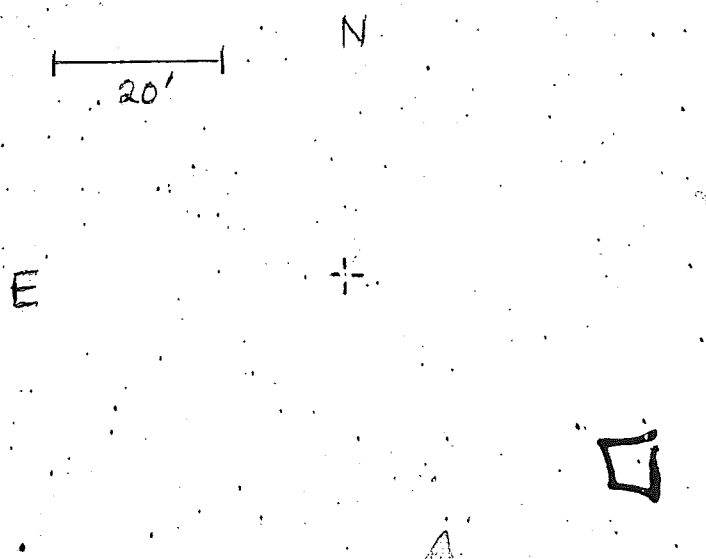


Figure 1a. SS Cygni near minimum.

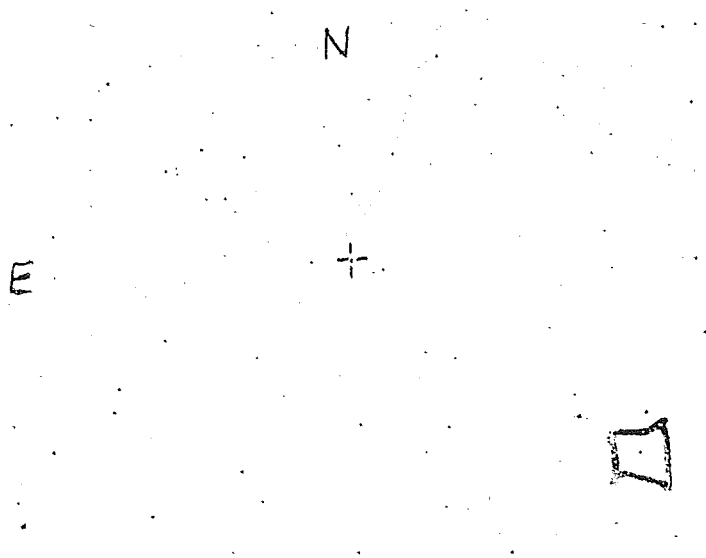


Figure 1b. SS Cygni in outburst.

The complete details of the discovery of SS Cyg may never be known, but with its discovery began a long period of visual observations so complete that an outburst of this well-studied star has never been missed.

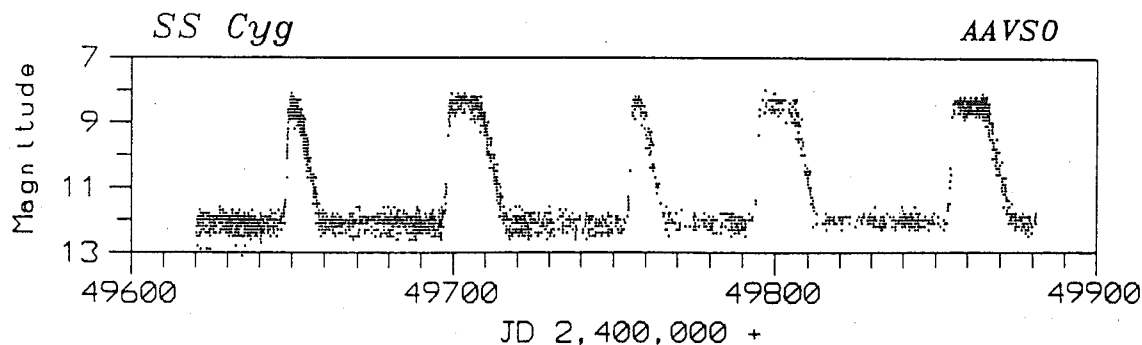
SS Cygni is one of the most famous variables in the sky. It is an eruptive variable of the *U Geminorum* type. Like most U Gem stars, SS Cyg is part of a spectroscopic binary system—a close system with a cool main-sequence star orbiting a white dwarf. The cool star loses matter from its surface, which accumulates in an accretion disk around the white dwarf. For as yet not well understood reasons, the disk becomes unstable and the disk material spirals towards the white dwarf triggering a small nova-like eruption in this dwarf nova system. (Hence the name of dwarf novae given to U Gem variables.) Every few weeks, on average, SS Cyg brightens from magnitude 12 to magnitude 8. At minimum, it can be observed with a small telescope. At maximum, it can be observed with binoculars.

SS Cyg is a spectroscopic binary consisting of dwarf G and subdwarf B spectral class components. The orbital period is 6 hours and 38 minutes; however, no eclipses are observed so the plane of the orbit must be considerably inclined to the line of sight. In other words, from our perspective here on Earth, one star does not pass in front of or behind the other. Spectroscopically, the system alternates between the spectrum of the dwarf G (5520 K) during minimum and the subdwarf B component during maximum. As the star rises to maximum, the spectrum changes progressively to that of the subdwarf B star (12,000K).

Visual observations of SS Cyg have become particularly important since it was discovered from satellite observations that SS Cyg is an extreme ultraviolet, and X-ray emitter. The satellite users depend on visual observers to tell them when the star is bright and active, and therefore worthy of further observation by satellite. Observations by amateur astronomers are necessary because the period of SS Cyg is not predictable. The rise to maximum is generally quite rapid, but by no means uniform. Some outbursts are quite short in duration. The light curve of other outbursts may be quite flat for several days at maximum. It then gradually slopes until a more rapid decline sets in. Occasionally there is a slight brightening before the actual maximum, and at other times there is a change from a steep slope to a more gradual one during the decline.

The minimum is the star's quiescent time. At times the light curve is quite flat and at other times quite irregular, with occasional rises in magnitude to at least magnitude 10.0 or brighter, preceding regular maximum.

SS Cygni has had over 700 outbursts since its discovery in 1896 and not one outburst has gone undetected. Statistical studies have shown that there are correlations between light curve characteristics such as the brightness and duration of outbursts and the interval between them. If you wish to study these correlations, you may—through the HOA Website—request long term data of SS Cygni from the AAVSO International Database.



Activity 12.5: Folded Light Curves of Star X and Delta Cep

1. If you have completed Core Activity 6.5 in Chapter 6 and calculated the period for Star X, you may now use your own estimated period for Star X as the period P , and take the brightest single observation as a rough estimate of the epoch t_0 , or time of maximum.
2. Using this period and epoch, and your data for Star X, follow the same procedure as in Core Activity 12.4. You will end up with a standard folded light curve of Star X.
3. Study the resulting diagram. Is your period accurate? Is there a lot of scatter? Is the epoch accurate? Is the maximum at (or near) phase 0?
4. Construct another standard folded light curve of Star X, but this time use the class average period as your period P . Are the folded light curves the same? Which period estimate do you think is more accurate?
5. If you have observed delta Cep, use your own observational data and estimate the period for delta Cep to construct a standard folded light curve. Was your period accurate?

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Core Activity 12.6: VSTAR

Let's say that you suspect that your data are fluctuating with some particular period. You could test this period by using it to construct a phase diagram. If the scatter in the resulting phase diagram is much less than the scatter in the data, then you have evidence that your data are fluctuating with that particular period.

You could even use this strategy to find an unknown period: simply test a *very* large number of periods by constructing phase diagrams. For a Mira-type variable, you might test periods from as short as 100 days to as long as 1000 days, in 0.1-day steps. So you would construct one phase diagram to test the period 100 days, another phase diagram to test the period 100.1 days, another for 100.2 days, etc., all the way up to 1000 days. That's a lot of phase diagrams!

It takes too long to do this by hand, but this kind of work is ideally suited to a computer. This particular period search method is one of the most common in astronomy. It is called the *analysis of variance*, also known as *AOV*. The VSTAR program will search for periods in your data, using AOV. If you find a likely period, it will construct a phase diagram for you.

Run the VSTAR program, and again select the star V Cas. Load the data from JD 2447000 to 2449000. Now hit the <F4> key (AOV (period)). It will ask you for the "number of bins" and suggest the answer 20; this is a good choice in this case, so just hit <ENTER>. You will see the "period analysis" menu.

Select option <F2> (frequency range). When it asks you for the "low frequency to test," enter 0.001, when it asks for the "high frequency to test," enter 0.01, and when it asks for the "frequency resolution," enter a 0 (which tells VSTAR to pick the resolution itself). Finally, when it asks "AOK?," answer "Y" and watch what happens.

VSTAR will test a lot of periods (frequencies), and for each period it tests it computes a *power* level. The power level is a measure of the likelihood that the data are periodic for a particular period. Power levels higher than 10 mean that the data *might* be periodic (or might *not*). As it computes, VSTAR draws a graph for you, showing power as a function of frequency. This is the most basic graph in period analysis: it is called a *periodogram*.

Possible periods show up as large power values, and on the periodogram plot they look like spikes. So when you see a spike in the periodogram, it represents a *possible* period. VSTAR saves the period of the ten tallest spikes in its "top-ten" list so you can access them later. After the periodogram plot is finished, hit the <F4> key. Now VSTAR shows you the top-ten list, giving the period and power level. Can you tell by looking which list entry goes with which spike on the graph?

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When you hit <F4>, VSTAR also asks which entries you want to delete. Enter “4-10,” so VSTAR will delete entries 4 through 10, leaving only 1, 2, and 3. These are certainly the most likely possible periods. Then give no answer to the “delete” question, and VSTAR will return you to the period analysis menu. Now hit the <F5> key (model the data). VSTAR asks which frequency to include. Enter “1” (to use the #1 period), and VSTAR will construct and display a phase diagram, using that period. VSTAR will ask if you want to “save to a file” (say “N” for no).

CAUTIONARY NOTE: When VSTAR shows you a phase diagram, it does not show two complete cycles, only one. Also, it does not put the maximum at phase zero; it chooses an epoch at random. So be prepared: if you see two complete cycles, it is not because it is a standard folded light curve! It is because your period is too long—you have gotten two cycles per period. Hit <F5> again, and this time choose period #2. You will see two complete cycles, because this is the wrong period! Period #2 is the twice the true period. Model the data again, using period #3, and you will see three complete cycles, because this is also the wrong period; it is three times too long.

When you fold the data using the correct period, you get a nice folded light curve of one cycle. When you fold the data using twice (or three times, four times, or any multiple of) the correct period, you also get a nice folded light curve, but of more than one cycle. So when VSTAR shows many possible periods, it is up to you to look at them, and decide which one is real.

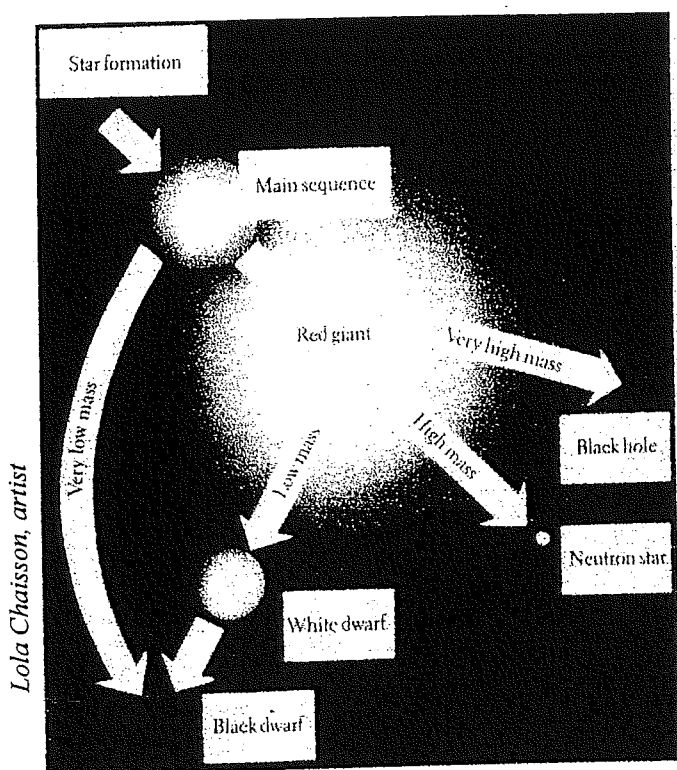
If the true period is P , then its multiples are $2P$, $3P$, $4P$, etc. Even though they give a high power level, they are not the true period, they are *aliases*. Since the frequency is $f=1/P$, the alias frequencies are $1/(2P)$, $1/(3P)$, $1/(4P)$, etc. So when the data are periodic with frequency f , you will get a spike at frequency f , and at frequencies $f/2$, $f/3$, $f/4$, etc. These alias frequencies are called *subharmonics*; it is a property of AOV that it gives high power levels not only for the true period (frequency), but also for its subharmonics. It is up to the analyst (you!) to decide which is real and which is an alias.

When you have decided which period is real, use that period to model the data. Now when you are asked whether you want to save to a file, say “Y” for yes. Then pick a file name for the saved information. VSTAR will create a file containing both phase and magnitude (just what is needed to plot the standard folded light curve). It will give you two complete cycles, and it will also estimate the maximum, choosing an epoch so that maximum is at phase zero: the standard folded light curve. See the VSTAR manual for the layout of this file. Plot the standard folded light curve for V Cas from JD 2,447,000 to 2,449,000. You will need to use a graphing program for this (there are too many data points to do it by hand!).

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Now repeat the entire procedure, but instead of using the data from JD 2,447,000 to 2,449,000, use the data from JD 2,440,000 to 2,442,000. Now you have *two* folded light curves, covering two different time periods. They show the *average* shape of the light curve during those time intervals. Are they the same? Is the average maximum brightness the same? Minimum? Is the *shape* of the average light curve the same?

Finally, a most interesting question: why does VSTAR show a spike at periods which are multiples of the true period (in other words, why does AOV respond to subharmonics)?



SPACE TALK

Mira stars are long-period, pulsating red giants of approximately the Sun's mass that have entered the final evolutionary stages of their existence and will eventually become white dwarfs. Miras have nearly exhausted the supply of hydrogen in their cores. Their cores are very dense and are composed mostly of oxygen and carbon (products of helium fusion). Just outside the core, a shell of hydrogen is still being converted to helium, and this layer of helium builds up on the surface of the core. Every few thousand years enough helium builds up and then ignites, creating even more carbon and oxygen. When the helium-

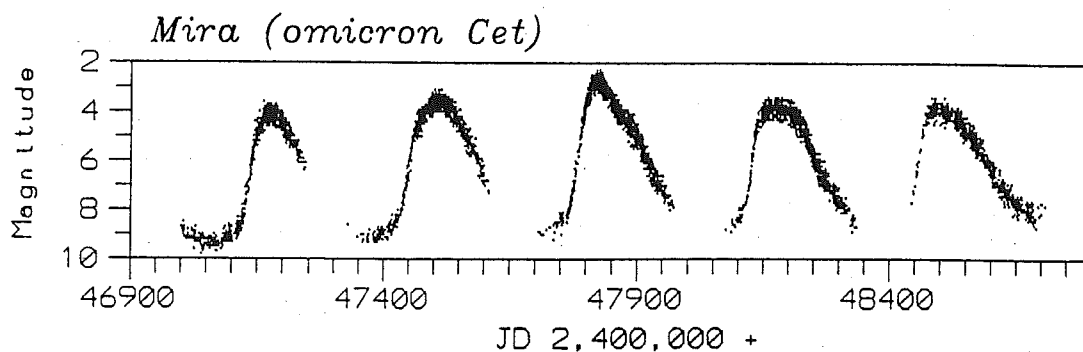
burning begins, the shell rapidly expands and the hydrogen-burning turns off. This is known as **helium flash**. When most of the helium is consumed, the flash ends, the shell shrinks, and hydrogen-burning resumes. This process can happen over and over again for 50,000 to 100,000 years before the outer layers are thrown off to form a planetary nebula. The core remains as a white dwarf, its nuclear fires finally out.

1996 marked the 400th anniversary of the discovery of the first Mira-type variable star. It is located in the constellation Cetus the Whale, and is known as Omicron (o) Ceti, or Mira, and was the very first known periodic variable star. David Fabricius, a clergyman and amateur astronomer in Friesland, Germany, noticed the "new" star in Cetus on August 13, 1596. Fabricius checked every star catalogue, atlas, and globe that existed and saw that the star was not listed. He observed Mira again at the beginning of September and watched it fade below naked-eye visibility during the middle of October. Fabricius assumed that his star was a nova similar to Tycho's nova discovered in Cassiopeia (Supernova of 1572). As a result, Fabricius did not check the star after it dimmed, because novae do not brighten more than once. He did not notice the star again until 1609. Fabricius might have studied his star more systematically with the recent development of the telescope, but unfortunately he met an untimely end when he was murdered by a member of his own parish in 1617. (He had recently announced from the pulpit that he knew which member of his parish had stolen one of his geese!)

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Mira was listed in Johann Bayer's 1603 star atlas *Uranometria* as an ordinary 4th-magnitude star labeled Omicron. Another German astronomer, John Holwarda, also from Friesland, discovered in the winter of 1638–39 that Mira had brightened, and realized that it would probably repeat the increase in magnitude. Every maximum since 1638 has been observed except for those times when Mira's apparent position in the sky appeared too close to the Sun to be seen, which occurs from April to the end of June each year. In 1667, Ismael Boulliau announced that Mira's variations were periodic and that the star brightened every 333 days. The period is 331.96 days, as published in the 4th edition (1985) of the *General Catalogue of Variable Stars* (GCVS). Johannes Hevelius, who named the star Mira ("the wonderful"), began observing it regularly in 1648, and in 1662 published a pamphlet about the star entitled "Historiola Mirae Stellae" ("Brief History of the Wonderful Star"). In 1926, Sir Arthur Eddington gave the correct explanation for the behavior of Mira-like variables—that these stars pulsate the same way Cepheid variables do, but with longer periods because of their more distended physical size and lower surface gravity.

Mira typically ranges in brightness from magnitude 9.3 to 3.4. In some cycles Mira brightens to a brilliant 2nd magnitude, and in other cycles barely reaches 5th magnitude. The brightest maximum on record occurred in November of 1779, when William Herschel observed Mira to be almost as bright as Aldebaran (magnitude 0.9). Maxima observed since 1906 by members of the AAVSO have ranged from 2.4 to 4.9, and minima have ranged from 8.4 to 9.7. The period also displays irregularities, with maxima arriving three weeks earlier or later than predicted. Unlike most Mira-type variables, omicron Ceti is a double star system. It has a 10th-magnitude companion, a hot dwarf called VZ Ceti. This companion was seen for the first time in 1923, although it had been detected by spectroscopic methods 5 years earlier. The irregularities in Mira's variations are obvious in the AAVSO light curve shown below, based on more than 17,000 observations by amateur astronomers over the past 25 years.



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A star pulsates because it is not in **hydrostatic equilibrium**: the force of gravity acting on the outer mass of the star is not quite balanced by the interior radiation pressure pushing outwards. If a star expands as a result of increased gas pressure, the material density and pressure decrease until the point that hydrostatic equilibrium is reached and then overshot, owing to the **momentum** of the expansion. Then gravity dominates, and the star begins to contract. The momentum of the infalling material carries the contraction beyond the equilibrium point. The pressure is again too high, and the cycle starts over again. The system acts as an **oscillator**. However, with loose atmospheric layers of gases, the oscillations get out of sync, or phase, with one another and set the stage for chaotic motions. Energy is dissipated during such pulsation (analogous to losses caused by friction forces), and eventually this loss of energy should result in a damping or lessening of the pulsations. The prevalence and regularity of pulsating stars imply that the dissipated energy is replenished in some way. This kind of statistical conclusion requires very long runs of data, such as those collected by the AAVSO.

Mira stars are not entirely predictable, and individual cycle lengths can be several weeks shorter or longer than the mean. Miras also undergo small, longer-lasting changes in their periods which are revealed by sophisticated statistical techniques. Subtle departures from regular behavior are characteristic of these stars. However, a handful of Miras go way beyond subtle changes, instead exhibiting extreme period changes that indicate radical physical changes within the interior of the star. One such star is R Hydrae, the third Mira-type variable discovered, located ~325 light-years away. (The second Mira-type variable discovered was chi Cygni in 1686.) R Hydrae missed discovery twice, once by Johannes Hevelius who recorded it in his star catalog simply as a 6th-magnitude star, and once by Geminiano Montanari, the Italian astronomer who worked at Bologna and discovered the variability of Algol in 1669. Montanari noticed R Hydrae at naked-eye brightness in April 1670, noted that it was not listed in Bayer's *Uranometria* star catalog, and added it to his copy by hand. Montanari's copy of *Uranometria* ended up in the possession of Giacomo Maraldi, who worked with his uncle, Giovanni Cassini, at the Paris Observatory. (Cassini discovered the division in the rings of Saturn which now bears his name.) Maraldi saw the handwritten notation in the star catalogue and began searching for the star in 1702, finally discovering it two years later at 4th magnitude.

The most remarkable aspect of R Hydrae is its slow but dramatic shortening of period, from 495 days during early observations to only 389 days during the last 60 years. For the first century of observation, R Hydrae's period decreased at a nearly constant rate, from 485 days/cycle around the year 1800 to 400 days/cycle around 1910. From 1923 to 1935, the period slowed to 415 days, then suddenly increased again. Since 1937, the period has remained constant at 389 days. The century of steady decrease in R Hydrae's period is consistent with theoretical calculations of what happens to a pulsating red giant after helium flash. R Hydrae is still recovering from such an event.

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Amateur variable star observers usually do not get to actually observe the dramatic changes that evolving red giants undergo. One exception is that of T Ursae Minoris, a red variable in the bowl of the Little Dipper. After decades of constant periodicity, the pulsations of T Ursae Minoris started to increase drastically, probably due to the earliest stages of helium flash. Before 1980, the period ranged from 310 to 315 days, but since 1980, it has decreased steadily to 274 days. If the theoretical models of stellar evolution are correct, T Ursae Minoris should continue shortening its period until it reaches a minimum period of 200 days (around the year 2030), after which its period will once again lengthen.

POSTER TALK: "Theoretical Glue": Understanding the Observed Properties of Miras with the Help of Theoretical Models

by Dr. Lee Anne Willson

Lee Anne Willson, Professor of Astronomy at Iowa State University, is an internationally-recognized expert on Mira variable stars. The following is an adaptation of a paper she presented at a special scientific conference on Mira stars sponsored by the AAVSO in 1996. The full text, including additional references and bibliography, can be found in *The Journal of the AAVSO*, Volume 25, No. 2, pages 99–114.

1. Introduction

Each observational study of Mira variables has as its goal to determine some quantities describing these stars. However, observations alone do not give us an understanding of what we are seeing. Theoretical models are needed both to connect what is observed to the qualities of the star, and to link the various measurements into one coherent picture of its nature. It is in this sense that a good model is a kind of "glue" holding the picture together.

Figure 1 illustrates the concept of "theoretical glue" by showing how the luminosity L , the radius R , and the effective temperature T_{eff} are related by the theoretical (and lab-tested) model of a blackbody "perfect radiator."

Such a perfectly radiating surface emits power per unit surface area that increases as the fourth power of T (in Kelvins), so a doubling of the temperature gives a 16-fold increase in the total amount of radiation (light, infrared, ultraviolet, X-rays and so on) coming from each patch of the surface. If stars were, in fact, ideal blackbodies, then their radiation would be completely known and the problem of relating L , R , and the temperature of the surface would be trivial. However, real stars are gas spheres; we can only see into the atmosphere on the average down to the apparent surface, the photosphere. We define the "effective" temperature T_{eff} as the temperature of a blackbody of the same size as the star that radiates the same total power, L . This gives the equation $L = 4\pi R^2 \sigma T_{\text{eff}}^4$, with $\sigma = 5.670 \times 10^{-8}$ watts per square meter per second per (Kelvin)⁴. Moreover, the stellar atmosphere is not all at one single temperature, and we see to different depths at different wavelengths; as a result, the spectrum we see has high and low spots compared with an ideal blackbody spectrum. Typically, the effective temperature is close to the gas temperature at the photosphere, but is not identical to it. We have to use a theoretical model atmosphere to relate effective temperature to the spectrum we see and to the temperature at the photosphere.

For Miras and other pulsating variables, even more than for most stars, the process of translating "what is actually observed" into "what the star is really like" can lead the incautious investigator astray. For example: To get the luminosity—the total power output, energy per second—we observe the visible part of the spectrum and, if we are lucky, also the near-infrared and perhaps the ultraviolet, in detail or using broad-band photometry. We then use a model of some sort to estimate how much

Radiation from an ideal Blackbody

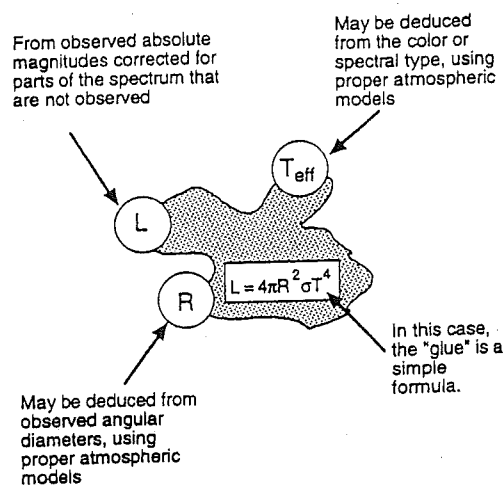


Figure 1. The definition of effective temperature (T_{eff}) illustrating the concept of theory as "glue."

light we are missing in the parts of the spectrum that we can't see, and finally we correct for distance. How good our final result is will depend on how good the model is that we use to fill in the "missing bits," as well as on how much of the spectrum we could actually observe and how accurately we know the distance. The expected (and sometimes observed) spectra of Miras are very far from the simplest case—a blackbody spectrum—so detailed models are essential. Worse, the molecules that produce some of the deepest spectral features in Miras are also found in Earth's atmosphere, and so we are selectively less likely to observe the depressed parts of the spectrum.

2. Classical model atmospheres

The calculation of a classical stellar atmosphere begins with a choice of stellar parameters—for example, composition, effective temperature, and surface gravity. The propagation of energy from the interior of the star through the atmosphere and into space is then calculated, taking into account the effects of the different atoms, ions, and perhaps molecules that can absorb and emit light. The result of a classical atmosphere calculation may include any or all of a predicted spectrum, a model for the pattern of brightness that you would see if you could get close to the star, and predicted values for the photometric colors. Such models play a key role in the determination of the luminosities of stars, and also in the derivation of their radii.

To get an estimate for the radius or diameter of a star, we may use an interferometer or a lunar occultation to get a pattern of fringes that may be interpreted by using a model for the brightness pattern on the star. Or, we may try to relate the appearance of the spectrum to the effective temperature T_{eff} using a detailed model atmosphere, and then deduce R from L and T_{eff} . If these methods give the same answer, it increases our faith that the model is close to describing what happens on the star.

To find the composition of the atmosphere, the line spectrum is analyzed using a stellar model atmosphere. Thirty years ago, most such calculations were made using some reference model atmospheres and looking for differences using methods such as the "curve of growth." Today, it is possible to carry out most analyses by making model atmospheres with a range of compositions and selecting the composition pattern that produces a spectrum that best matches the observations.

Figure 2 illustrates how a classical stellar model atmosphere glues together observable and non-observable quantities for stars. In a classical model atmosphere there is no net outflow of matter—no stellar wind—and there are no systematic motions, such as one might get from pulsation. Obviously, this is not going to work perfectly for modeling Miras! Also, in classical atmospheres, each part of the atmosphere is in radiative equilibrium—meaning that the radiant energy flowing into a sample volume of the gas per second exactly equals the radiant energy flowing out of the same sample volume per second. In more modern model atmospheres, other forms of energy are also considered, and energy is allowed to shift from one form to another—for example, from soundwaves to radiation. There are still relatively few models, however, that include dynamical effects and outflows.

Model stellar atmospheres

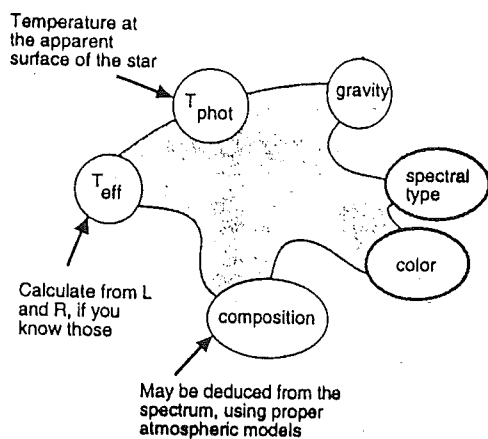


Figure 2. Classical model atmospheres as "glue" for the colors and spectra.

One of the important ingredients in a stellar atmosphere model—whether classical or modern—is the surface gravity $g = GM/R^2$, where

G is the gravitational constant, and M is the mass of a pulsating star. This combination of M and R turns out to be important for the spectrum because higher gravity compresses the atmosphere more. One can deduce a gravity by computing synthetic spectra for models with a range of surface gravities, and then picking the model whose spectrum best matches the star, as long as the model does produce a good match. This works reasonably well for most non-variable stars, but does not do a good job with the variable ones. Other methods are needed for these, at least for now.

3. Glue from pulsation and evolution studies

In principle it should be easier to derive a value for the surface gravity for a pulsating star, because the material in the atmosphere is moving in response to gravity during much of the cycle. Thus, one might observe the change in velocity over some interval of time and estimate $\Delta v / \Delta t = g$. Because the excursion in radius is large (so g is not the same at all parts of the path) and because pressure forces are also important, the above method typically underestimates g by a factor of five or so. Instead, a dynamical atmosphere model needs to be used to interpret the result. Also, you need a good radiative transfer model to be able to interpret the observed

Doppler shift in terms of the motions of parts of the atmosphere, because only part of what you see is moving towards or away from you. To get a meaningful Δv from the Doppler shift requires a fairly detailed model for the atmosphere, and this correction is still rather rough for most variable stars.

There is another way to get a combination of M and R for pulsating stars. A given star is usually able to pulsate only in one or a small number of modes, each with a distinct period associated with it. Detailed models for the interior of a pulsating star can be analyzed to reveal the period(s) that are possible, and these can be related through formulae such as (for example), $P = a R^b M^c$, as is illustrated in Figure 3. (Usually b is between 1.5 and 2, and c is between -0.5 and -1.)

There are assumptions that go into this kind of modeling that need to be tested more thoroughly than has been possible so far: for example, the period of pulsation of a star pulsating at full amplitude may not be the same as the period derived looking at very small pulsations in a model for a static star.

A PMR relation is often used to estimate the mass of a pulsating star, M , given its radius, R (which may have been derived from L and T_{eff} or from angular diameter measurements), assuming that one knows the mode of pulsation. It can otherwise be used to determine the pulsation mode(s), if one is confident of M and R from other measures. For most classes of stars this is relatively easy to do, and the results are consistent with whatever else one knows about the stars. However, for Miras the radii and masses are still sufficiently uncertain that this method does not even yield an incontrovertible result about the pulsation mode, much less useful estimates for their masses.

Pulsation testing of model stars

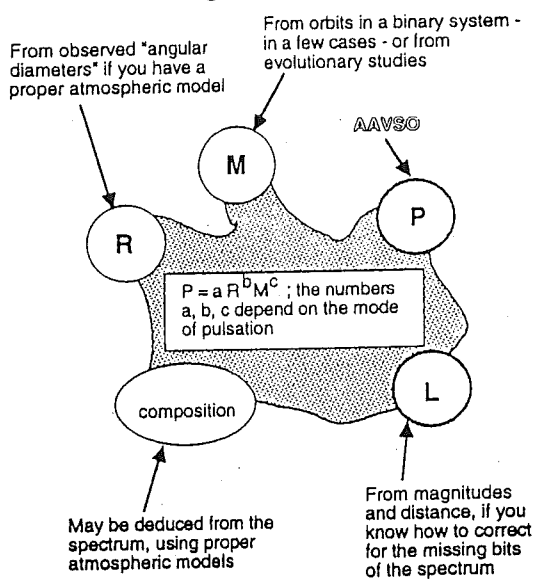


Figure 3. Pulsation periods from model stellar interiors connect evolutionary models to stellar parameters.

4. Models for stellar evolution

The most important "theoretical glue" in stellar astronomy is the study of how stars evolve. Starting with some composition (assumed to apply throughout the star) and a mass, M , a model is found that obeys relevant physical equations and is in hydrostatic equilibrium. For all but the lowest-mass stars the model will include energy generation by nuclear reactions in the core. These reactions modify the composition at the center, so some time later the star's structure will be a little different and its L and R may also be a little different. By building a sequence of static models that are related by the condition that the change of composition comes from the nuclear reactions, one can model the evolution of the star.

In most evolutionary calculations the mass M is not allowed to change with time, although there are times in the life of a star when the mass decreases as the result of mass loss from the surface. The change in mass that comes from the conversion of mass to energy in the nuclear reactions is almost always small enough to ignore. One time when the mass loss is particularly important is the Mira stage, and this fact is a major reason why Mira models are not yet in a settled state.

Dynamical atmospheres for pulsating stars

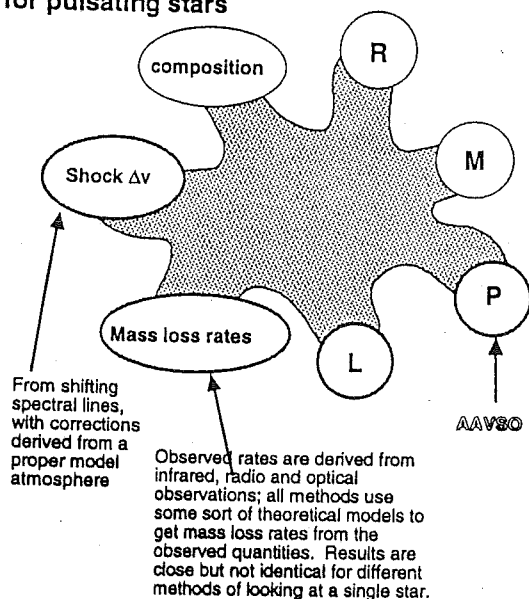


Figure 5. Dynamical model atmospheres are required for pulsating stars, such as Mira variables.

Evolving stellar models

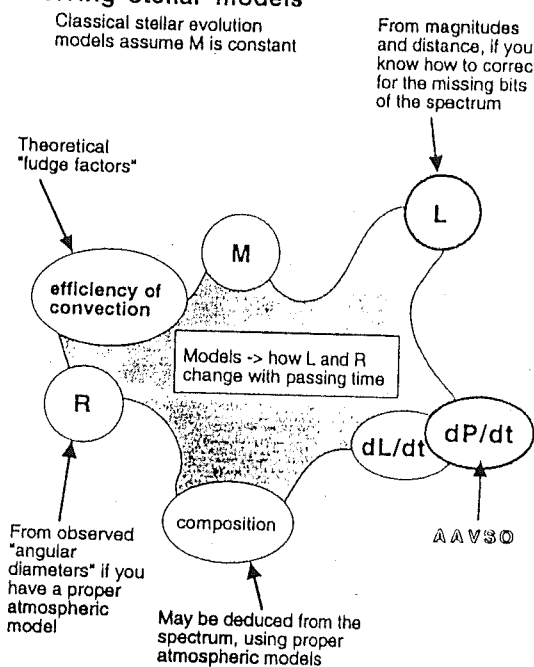


Figure 4. Evolutionary models link stars in different evolutionary states as well as relating stellar properties and (occasionally) rates of change of those properties.

Since much of stellar evolution proceeds at constant mass, and since L and T_{eff} are the easiest quantities to estimate directly from observations, we traditionally plot tracks for constant mass stars in a diagram of L versus T_{eff} , one variant of the Hertzsprung-Russell diagram. One may then think of evolutionary tracks as linking L , M , T_{eff} or R , initial composition, and age for the star (Figure 4).

In addition to the problem of how to include mass loss in a realistic way, evolutionary models also suffer from our lack of detailed understanding of convection in stars; of rotation inside stars; of the effects of magnetic fields in stars; and so on. Most astronomers assume that these effects will turn out to be small, but others would not be surprised to learn that some of them affect the "big picture."

5. Dynamical models for the atmospheres of pulsating stars

If we know how a star is pulsating, then we can model the response of the outer parts of the star (the atmosphere) to this pulsation. Figure 5 shows the connections that can be made this way. In practice, L , M , and T_{eff} or R are assumed; also a pulsation period P is assigned (using a PMR relation) and the bottom of the atmosphere is made to move in and out with period P .

Dynamical models require some understanding of the interaction between the gas and the radiation. The pulsation generates waves that compress the gas, heating it. It then cools by radiating away energy, and also by expanding. Depending on the density of the gas, the conversion of internal energy into radiation may be fast (compared with the pulsation time) or slow. Where it is fast, the material cools to roughly the equilibrium temperature that it would have in a static model, and then as it expands it may be refrigerated below the temperature it would have in the static case.* Where the density is lower, the cooling is less efficient; there, the temperature may never fall as low as the equilibrium temperature. Some dynamical model results are very sensitive to the treatment of these processes; mass loss is one example. Since the details of how the gas emits or absorbs radiation at low density involve many non-equilibrium chemical processes, this is definitely one of the frontier areas in dynamical atmosphere modeling.

A detailed treatment of the interaction between gas and radiation—the radiative transfer problem—is also required in order to synthesize the spectrum and colors that would be observed, as well as the light curve. So far, there is no model for Miras or other pulsating stars that includes enough detail to do this effectively. However, dynamical models that are now available provide important insight into the motions of the atmospheres and the mass loss rates that result. For example, Bowen's models (Bowen 1988, 1990) have atmospheric motions and conditions that match what we deduce from observations—shocks with velocity amplitudes of 20 to 30 km/s, warm regions in some, dust formation in others, and so on. In fact, the success of dynamical models in matching velocity variations observed in the infrared CO lines is the best evidence we have about the mode of pulsation of these stars—fundamental mode models match well, but overtone models (with larger radius at a given P) are quite far from matching, as was first noted nearly 20 years ago (Hill and Willson 1979).

Dynamical model atmospheres with radiative transfer (etc)

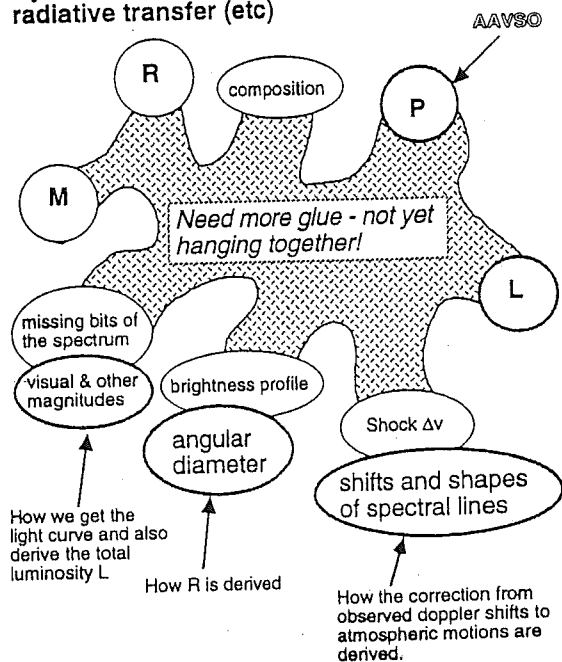


Figure 6. Ideally, radiative transfer, non-equilibrium chemistry, and detailed hydrodynamics are included in the models. In practice, no one model yet includes all the details that are needed to reproduce all the observations.

* Since the density is highest just after the gas is compressed, there is a region in the atmosphere where it can lose energy to radiation immediately after compression but has a harder time regaining energy near the end of its expansion. We can describe this approximately by saying that the shock is nearly isothermal—it returns to the radiative equilibrium temperature quickly—but the expansion between shocks becomes nearly adiabatic—without gain or loss of energy.

6. Some results of recent "glue" production

Bowen's latest grid of dynamical atmosphere models is a collection of models that are constrained by stellar evolution calculations: once L , M , and initial composition are chosen the evolutionary calculations are used to derive R , and then a theoretical PMR relation gives P . This single step of requiring the stars to fall on a single set of evolutionary tracks turns out to make quite a big difference in the way that the mass loss is understood to develop. The choice of which tracks to use is not so important as is the fact that using tracks forces certain relationships between models: For a given mass, as a star increases in luminosity it also increases in radius with (slightly) decreasing T_{eff} . Two stars with the same L and different masses will be separated in T_{eff} or R . Two stars of the same L and M but different composition will also be separated in T_{eff} or R : lower metallicity stars are hotter and smaller at a given L .

Bowen's models, constrained in this way, predict mass loss rates that are very sensitive to stellar L , R , and M . Since all of these parameters vary in a predictable way along a given evolutionary track, we can display the results as mass loss rate $\dot{M}$ versus L for a given mass, where for a given metallicity L and M together also determine R , T_{eff} , and P as well. Figure 6 shows the result of these calculations for stars whose composition matches that of the Sun.

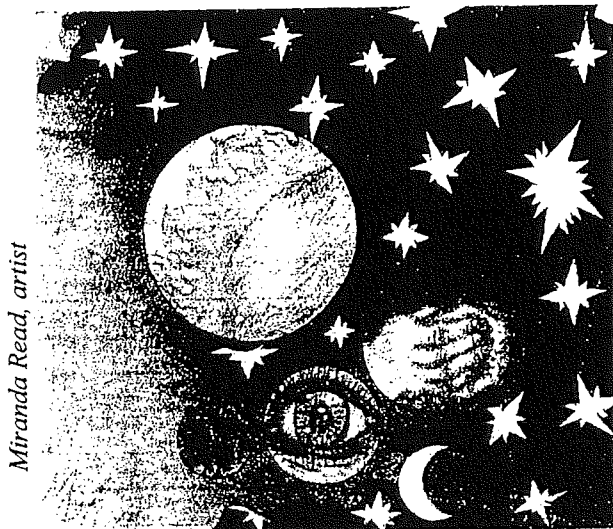
7. Conclusion: where we need some new glue

Many pieces of the puzzle of Miras and other pulsating variable stars are well-glued together by these theoretical calculations, but some very basic properties remain "unglued." The outstanding problem in the case of the Miras remains the determination of their absolute sizes; here, uncertainties of factors of two or more are still a problem. Clearly we need more and better "glue"—new models that incorporate more of the physics that we already know is important in producing what is observed.

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Chapter 13: Variable Stars and O-C Diagrams



Miranda Read, artist

Introduction

In the previous chapter we stated that a *perfectly periodic* system repeats exactly the same behavior, over and over again. Every cycle is precisely like every other cycle. Some variable stars actually behave this way: Cepheid variables, for example, may repeat precisely the same brightness variations for thousands of cycles. Eclipsing binaries also sometimes repeat thousands of cycles, with exactly the same period every time.

Other variables are not quite so reliable. While they do go through an almost never-ending series of ups and downs, each cycle is a little different from every other cycle; the period will be slightly longer or shorter, and the maximum and minimum magnitudes will be slightly brighter or fainter. You probably noticed in the last chapter that the light curve of the Mira-type variable V Cas showed slight differences from cycle to cycle. It keeps brightening and dimming, so it definitely appears to be periodic, but it is not *perfectly periodic*.

In fact, all Mira-type variables behave this way. They are periodic, because they keep repeating cycles over and over again. But they are not perfectly periodic, because every cycle is a little different. For Mira-type variables, the differences from cycle to cycle are small. The period, for instance, will be a little different for each cycle, but is usually within 10% of its average value. The amplitude of each cycle (the difference between minimum and maximum brightness) will usually be within 20% of its average value. Each cycle in the light curve looks a bit different, but they all have many similarities as well, and they are usually close to the “average shape” of the light curve.

Another class of variables, known as the *semiregular variables*, show even greater differences from cycle to cycle. Not only are their periods not perfectly periodic, but these stars also sometimes “switch” from one period to another (a process known as *mode switching*). Their amplitudes change dramatically: they may suddenly increase their variability, or they may stop varying altogether (but when they do, they usually start up again soon after).

It is easy to see the differences from cycle to cycle in the light curves of Mira- and semiregular-type variables. The light curves of Cepheids show what appear to be identical cycles. But if we watch Cepheids long enough—for tens of thousands of cycles—we can detect very slight changes in their cycles as well. They too can show changes in the cycle shape, the amplitude, and the period. The differences are still there: even the Cepheids are not perfectly periodic.

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Pulsating Stars

When the first Mira-type variables were discovered, it was a mystery why they were varying at all. After all, most stars, like our own Sun, are quite stable, not variable (even the Sun varies a *little* bit, but surprisingly little). In the early part of this century, the famous English astronomer Arthur Eddington studied the problem carefully. Most known ways that stars can vary could be eliminated from consideration. For example, Mira-type variables were not exploding like supernovae: their fluctuations are too regular for that. And they were not eclipsing like the eclipsing binaries: their fluctuations are not nearly regular enough for that.

Eddington went back to the basics. Stars glow like the filament of a light bulb for the same reason—because they are hot. The light output of a star depends mainly on two things: its surface temperature (how bright the light bulbs are) and its size (how many light bulbs are burning). The apparent visual magnitude of Mira itself is 100 times brighter at maximum than at minimum. To be so much brighter, it would either have to be hotter, or bigger, or both.

By studying the spectra of stars, we can get a good estimate of their temperatures. The spectrum of Mira throughout its cycle does change, and in fact Mira will show temperature changes while it fluctuates, but these temperature changes are just too small to explain the tremendous increase and decrease in brightness. That leaves only one possibility, said Eddington: Mira changes its brightness by a large amount every cycle because Mira changes its size by a large amount every cycle. Mira (and all Mira-type variables) fluctuate because they are expanding and contracting, growing and shrinking. The star is literally “vibrating.” Variables that do this are known as pulsating variables.

We now know that there are many types of pulsating variables. Cepheids, for example, are very regular pulsating variables, usually with periods of a few days or more. Mira-type variables also pulsate, but their periods are almost always more than 100 days, and their pulsations are much more irregular from cycle to cycle than those of Cepheid variables. Semiregular variables pulsate with periods from as little as 30 days to over 1,000 days, and their pulsations are even more irregular than those of Mira-type variables.

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Period

Eddington's "back-to-basics" approach was appropriate. After all, the stars are so far away that we have to stretch ourselves just to uncover their basic physical properties. The most important physical parameters of a star are its size (the radius) R , its surface temperature T , and its mass M .

Unfortunately, we cannot measure these basic quantities directly. The easiest to estimate is the temperature. The spectrum of the star acts almost like a thermometer to give us a good estimate of the temperature, and other clues enable us to refine this estimate. The radius can be quite difficult to determine, because the stars are so far away that we cannot really see their images directly (except in a few special cases). Still, if we know the star's distance from earth, its brightness, and its temperature, we can get a reasonable size estimate. Of all the "basic parameters," the most difficult to estimate is the mass: too often we simply have no clues, and our best mass estimate is likely to be very unreliable.

There are exceptions. Binary stars orbit each other, and their orbits are determined by the laws of gravity. The strength of gravity depends on mass, so if we know all the details of the orbital motion of a binary star, we can get a very good mass estimate. That is one of the reasons eclipsing binary stars are so important: the details of their orbital cycles enable us to determine stellar masses. Unfortunately, this can be applied only to a few stars.

Another exception is pulsating stars. The period of a pulsating star depends mainly on its basic physical parameters of size, temperature, and mass. So if we know the period, we have one more clue to help us estimate mass. In fact, the period of a pulsating star can give us clues about its mass, the strength of its gravity, and sometimes even the reactions taking place in the star's interior.

According to our theories of stellar structure, if a star is pulsating, its period in most cases will be stable. The period may change from cycle to cycle (as with Mira-type variables), but the *average* period over many cycles will remain the same. For the average period to change, we would have to change the star's mass, size, or temperature (none of which is very likely), *or* we would have to change the internal workings of the star. Such changes do occur, but they usually happen only at important stages in the star's life cycle. So when a pulsating star shows a change in its average period, it usually means that the star is undergoing an evolutionary change in its behavior, moving from one stage in its life cycle to the next.

We see that the period of a variable star is one of the most revealing aspects of its variations. It is something that we can obtain through careful observation. The average period gives us vital clues about a star's basic properties, including mass and gravity. Any period *changes* are a warning that the star may be entering an important new stage of

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its development. And this is true not just for pulsating variables: for any periodic variable star, the period is one of the most important and most informative of its observable parameters. Because of this, studying the periods of variable stars, and especially any changes in their periods, is an especially important part of the analysis of variable stars.

O-C

One very useful tool for finding period changes, and one which is very popular with astronomers, is to compute what is called “*O-C*” (“*O* minus *C*”). It is based on the following idea: if a star is perfectly periodic, then every period is exactly the same. In that case, we can predict its cycles in advance. If one maximum occurs on, say, JD 2,450,000, and the period is precisely 332 days (and never changes!), then the next maximum will occur on JD 2,450,332. This is not just guesswork: it is based on the way periodic systems behave. This is how scientific predictions are made: by combining a precise *theory* of behavior (perfect periodicity) with accurate parameters (the epoch, or time of maximum, and the period) determined from precise observations (made by careful variable star observers), we can predict the behavior of a star in advance. Then we can perform what may be the most powerful test in all of science: we can compare our predictions with future observations.

If the star is perfectly periodic, has a maximum at time t_0 (the *epoch*), and the period is P , then we know that the *next* maximum will occur at time $t_0 + P$. The next maximum after that will be at time $t_0 + 2P$, then next at $t_0 + 3P$, etc. In fact, if we choose t_0 , our *epoch*, to be the time of maximum for cycle number *zero*, then the *computed time of maximum* for any cycle number n , which we can call C_n , is easy to calculate:

$$C_n = t_0 + n P.$$

With this one formula, we can compute the times of all maxima, past, present, and future.

Of course, these times are correct only if the system is perfectly periodic. In addition to the computed times of maximum C_n , we can also directly observe the star to estimate the *observed time of maximum* for cycle number n , which we will call O_n . You have already done this in previous chapters, estimating the time of maximum either by eye, or by fitting a polynomial to the light curve data. We are now ready to compare theory (the computed times C_n) to observation (the observed times O_n), by simply taking the difference between the observed and computed times of maxima. These are the “*O-C*,” or “observed minus computed” values. For each cycle number n , we have $(O-C)_n = O_n - C_n$.

After we have determined the *O-C* values, we can plot *O-C* as a function of cycle number n . This gives us a powerful tool for period analysis: the *O-C diagram*.